



丘成桐数学科学中心
YAU MATHEMATICAL SCIENCES CENTER



Stable homotopy groups of spheres: Theories and Computations, where to now?

March 9-13, 2025
Room A-103, TSIMF

ORGANIZERS

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Foling Zou(邹佛灵), Chinese Academy of Science

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Zhouli Xu(徐宙利), University of California, Los Angeles

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About the conference

球面稳定同伦群：理论与计算的现状与展望

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Date

March 9-13, 2025

Venue

Room A-103, TSIMF

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Abstract

The stable homotopy groups of spheres have been a central and enduring topic in algebraic topology, playing a key role in our understanding of the deep structure of spaces. Over the decades, substantial progress has been made in both theoretical and computational aspects of these groups. Classical tools such as spectral sequences, especially the Adams and Adams-Novikov spectral sequences have provided a robust framework for advancing the theoretical understanding of stable homotopy groups. However, many questions remain unanswered.

This workshop will focus on the problem of computing stable homotopy groups, including both history and the recent advances in this subject. We will begin by providing an overview of the history and foundational concepts underlying the study of stable homotopy groups of spheres. We will also explore how these groups are connected to broader areas in algebraic topology and mathematics, such as cobordism theory, K-theory, and motivic homotopy theory.

The computational aspect has seen recent breakthroughs, particularly through the use of modern tools like motivic and equivariant homotopy theory, for example, the recent result by Lin, Wang and Xu. We will survey some of the latest results in these areas, highlighting the role of computational techniques, including the use of motivic tools, higher chromatic tools and spectral sequences, which have pushed the boundaries of what can be computed.

Despite these advancements, significant challenges remain in understanding the full structure of the stable homotopy groups of spheres. We will address some open problems in the field. Finally, we will outline possible future directions: where does the future of stable homotopy theory lie, and how might we move closer to a complete understanding?

Description of the aim

This workshop aims to gather leading experts and early-career researchers to explore the latest advances in the field of stable homotopy theory, with a particular focus on the calculation and structure of stable homotopy groups of spheres. As one of the most fundamental problems in algebraic topology, understanding these groups has led to deep insights across multiple areas of

mathematics, including cobordism theory and K-theory.

Thematic Focus:

The workshop will revolve around the following thematic areas:

1. Computational Techniques and theoretical developments: Recent progress in computational methods, particularly the use of motivic and equivariant homotopy theory, has led to new insights into the stable homotopy groups of spheres. Sessions will focus on motivic homotopy theory, computation-assisted spectral sequences computation, and other advances in computational tools, offering a comprehensive overview of the current state of the art.

2. Interactions with Other Fields and Open Problems: The workshop will emphasize the intersections of stable homotopy theory with other mathematical areas. We will address some open problems in the field and outline possible future directions.

Distinguished Lectures (tentative):

The workshop will feature special lectures on Adams spectral sequence computation techniques in the recent breakthrough work of Weinan Lin, Guozhen Wang and Zhouli Xu. whose pioneering work on Adams spectral sequence computation has largely extend our understanding of the stable homotopy groups.

In addition to these distinguished lectures, the workshop will include interactive problem-solving sessions, collaborative working groups, and open discussions on new computational challenges and conceptual developments. The goal is to foster collaboration, encourage the exchange of ideas, and inspire further research in stable homotopy theory and its related fields.

Schedule

Time&Date	March 9 Sunday	March 10 Monday	March 11 Tuesday	March 12 Wednesday	March 13 Thursday
7:30~8:30	Breakfast				
9:30-10:30	Cheng Li	Shangjie Zhang	David White	Peter May	Yu Zhang
10:30-11:00	Break and Refreshment				
11:00-12:00	Yueshi Hou	Yuchen Wu	Sihao Ma Group Photo	Shangjie Zhang 2	Yuchen Wu 2
12:00-14:00	Lunch Break (120 minutes)				
14:30-15:30	Group Discussion	Group Discussion	Free Afternoon 13:30-17:00	John Greenlees	Free Afternoon
15:30-16:00	Break and Refreshment			Break and Refreshment	
16:00-17:00	Group Discussion	Group Discussion		Alex Waugh	
17:30-19:00	Dinner			Banquet 18:00-20:00	Dinner

March 9, 2025 - Sunday

Time(日期)	Name(报告人)	Title(报告题目)
7:30~8:30	Breakfast	
9:30-10:30	Cheng Li(李铖)	basics of synthetic homotopy theory
10:30-11:00	Break and Refreshment	
11:00-12:00	Yueshi Hou(侯悦石)	synthetic Adams spectral sequence vs λ -Bockstein spectral sequences.
12:00-13:30	Lunch Break	
14:30-15:30	Group Discussion	
15:30-16:00	Break and Refreshment	
16:00-17:00	Group Discussion	
17:30-19:00	Dinner	

March 10, 2025 - Monday

Time(日期)	Name(报告人)	Title(报告题目)
7:30~8:30	Breakfast	
9:30-10:30	Shangjie Zhang(张尚杰)	Hidden extensions in Adams E_r page
10:30-11:00	Break and Refreshment	
11:00-12:00	Yuchen Wu(吴雨宸)	the generalized Leibniz rule and the generalized Mahowald trick.
12:00-13:30	Break and Refreshment	
14:30-15:30	Group Discussion	
15:30-16:00	Break and Refreshment	
16:00-17:00	Group Discussion	
17:30-19:00	Dinner	

March 11, 2025 - Tuesday

Time(日期)	Name(报告人)	Title(报告题目)
7:30~8:30	Breakfast	
9:30-10:30	David White	Gaunce Lewis's work on Mackey Functors
10:30-11:00	Break and Refreshment	
11:00-12:00	Sihao Ma(马思浩)	Towards the $K(2)$ -local homotopy groups of $RP^2 \wedge CP^2 \wedge HP^2$ at $p=2$
	Group Photo	
12:00-13:30	Lunch Break	
14:30-15:30	Free Afternoon 13:30-17:00	
15:30-16:00		
16:00-17:00		
17:30-19:00	Dinner	

March 12, 2025 - Wednesday

Time(日期)	Name(报告人)	Title(报告题目)
7:30~8:30	Breakfast	
9:30-10:30	Peter May	Homotopical Beck monadicity and iterated loop space theory
10:30-11:00	Break and Refreshment	
11:00-12:00	Shangjie Zhang(张尚杰) ₂	Mahowald invariant and its equivariant generalizations
12:00-13:30	Lunch Break	
14:30-15:30	John Greenlees	The landscape of rational G -spectra for a compact Lie group G .
15:30-16:00	Break and Refreshment	
16:00-17:00	Alex Waugh	Equivariant Power Operations
17:30-19:00	Banquet 18:00-20:00	

March 13, 2025 - Thursday

Time(日期)	Name(报告人)	Title(报告题目)
7:30~8:30	Breakfast	
9:30-10:30	Yu Zhang(张宇)	Computing the Adams E ₂ -Page for Odd Primes
10:30-11:00	Coffee Break	
11:00-12:00	Yuchen Wu(吴雨宸) ₂	A synthetic approach for differentials in equivariant slice spectral sequences
12:00-13:30	Lunch Break	
14:30-15:30	Free Afternoon	
15:30-16:00		
16:00-17:00		
17:30-19:00	Dinner	



Titles and Abstracts

basics of synthetic homotopy theory

Cheng Li(李铖)

University of California San Diego

Following Lin-Wang-Xu section 3, I will introduce the HF2 synthetic homotopy category and some of its basic properties computationally.

synthetic Adams spectral sequence vs λ -Bockstein spectral sequences.

Yueshi Hou(侯悦石)

University of California San Diego

In this talk, I will discuss the above two spectral sequences of various synthetic lifts of η^3 . In particular, the λ -Bockstein spectral sequence is always rigid, whereas the synthetic Adams spectral sequence always "looks like" the classical Adams spectral sequence in the bigraded diagram.

Hidden extensions in Adams E_r page

Shangjie Zhang(张尚杰)

University of California San Diego

Following Lin-Wang-Xu section 2-5, I will introduce the extension spectral sequence and how it helps with defining hidden extensions in the classical Adams E_r page for $r \leq \infty$, focusing mainly on examples. I will also talk about the effect of a crossing of extensions, a situation we want to avoid in general to apply the generalized Leibniz rule and the generalized Mahowald trick.

the generalized Leibniz rule and the generalized Mahowald trick.

Yuchen Wu(吴雨宸)

University of California San Diego

Following Lin-Wang-Xu section 6, I will introduce the generalized Leibniz rule and the generalized Mahowald trick and how they help with concrete computations in practice.

Gaunce Lewis's work on Mackey Functors

David Andrew Victor White
Denison University

This talk will summarize what is known about the algebra of Mackey functor rings, including (projective/injective/flat/simple) modules, (prime/principal/maximal) ideals, the prime ideal spectrum, tensor products, discrete valuation rings, integral domains, division rings, fields, etc. In several places, additional care is required as compared to the classical ring-theory context. The main point of the talk will be to provide a road map to a book that Gaunce Lewis was writing when he died in 2006, that Peter May, Mike Hill, and I are working to finish in the near future. This book contains change of group results, change of ring results, induction results, and localization results. Lastly, if there is time, I will briefly discuss the connection to equivariant homotopy theory, via a bridge that shows which structures on equivariant ring spectra (and their modules) gives rise to the corresponding structure in Mackey functor algebra, including new results about ideals of equivariant ring spectra.

Towards the $K(2)$ -local homotopy groups of $RP^2 \wedge CP^2 \wedge HP^2$ at $p=2$

Sihao Ma(马思浩)
University of Notre Dame

A finite spectrum can be reassembled from its $K(n)$ -localizations. While the homotopy groups of the $K(2)$ -local sphere at the prime 2 have not been computed completely, there has been some progress on the computation of the $K(2)$ -local homotopy groups of finite spectra using the algebraic duality spectral sequence and the topological duality spectral sequence. In this talk, I will talk about the computation of the algebraic duality spectral sequence and the topological duality spectral sequence associated with $RP^2 \wedge CP^2 \wedge HP^2$.

Homotopical Beck monadicity and iterated loop space theory

Peter May
University of Chicago

I will give an overview of three papers in progress by Hana Jia Kong, Foling Zou, and myself. I will discuss the homotopical version of Beck monadicity for a pair of adjoint functors and the axiomatic refinement of this theory when given monads with special properties. Time permitting, I will also talk about applications using composite adjunctions to orbital presheaves and to multiplicative infinite loop space theory.

Mahowald invariant and its equivariant generalizations

Shangjie Zhang(张尚杰)²
University of California San Diego

In this talk, I will review the classical Mahowald invariant and some of the computations and applications. The C_2 equivariant interpretation of Bruner-Greenlees allows us to define the generalized equivariant Mahowald invariant for larger groups. I will discuss some of the computations for the group C_{p^n} and their applications. This contains work joint with William Balderrama, Eva Belmont, Yueshi Hou, and Zhouli Xu.

The landscape of rational G-spectra for a compact Lie group G.

John Greenlees
University of Warwick

The talk will describe the general form of the category of rational G-spectra from two different points of view.

(1) There is a precise description of the associated graded of a finite filtration, based on sheaves over the Balmer spectrum (joint work with S. Balchin and T. Barthel).

(2) There is an approximate description of the assembled category: it is a product of blocks each dominated by a subgroup H. The block dominated by H is controlled by an integral representation of the finite group $\pi_0(H)$; to a first approximation it is a tensor product of pieces corresponding to the isotypical summands of the rationalization of the integral representation.

Equivariant Power Operations

Alexander Waugh
University of Washington

In this talk, I will introduce a general method for constructing stable operations which can be applied to the homotopy groups of any highly commutative G-spectrum which is a module over a fixed ring spectrum. Such operations are equivariant analogs of the classical Kudo-Araki-Dyer-Lashof operations. This construction recovers all of the operations in the case where the group of equivariance is trivial or has order two. Using this method, I will construct new nonzero operations for every finite group. This is joint work with Prasit Bhattacharya and Foling Zou.

Computing the Adams E₂-Page for Odd Primes

Yu Zhang(张宇)
Tianjin University

In this talk, I will present my ongoing joint work with Weinan Lin on algorithmic computations of the Adams spectral sequence E₂-page for odd primes. By developing a generalized Gröbner basis framework, we determine both the additive basis and complete multiplicative structure of the E₂-page in bounded inner degree ranges ($t \leq t_{\max}$).

A synthetic approach for differentials in equivariant slice spectral sequences

Yuchen Wu(吴雨宸)²
University of California San Diego

The theory of synthetic spectra has led to significant breakthroughs in the computation of Adams-type spectral sequences. The formalism underlying these synthetic techniques is also applicable to the study of other spectral sequences arising from filtered spectra. In this talk, I will employ synthetic methods to compute some differentials in equivariant slice spectral sequences for the Hill—Hopkins—Ravenel theories.



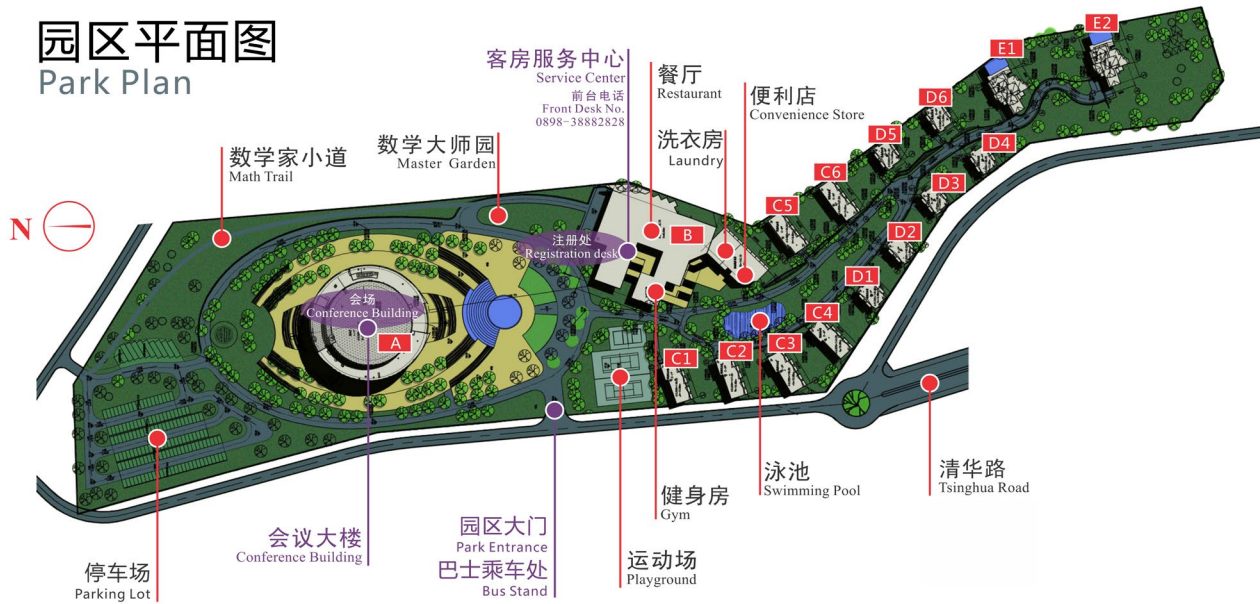
Welcome to TSIMF



The facilities of TSIMF are built on a 23-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 29,000 square meter that includes state-of-the-art conference facilities (over 10,000 square meter) to hold many international workshops simultaneously, two reading rooms of library, a guest house (over 10,000 square meter) and the associated catering facilities, a large swimming pool, gym and sports court and other recreational facilities.

Management Center of Tsinghua Sanya International Forum is responsible for the construction, operation, management and service of TSIMF. The mission of TSIMF is to become a base for scientific innovations, and for nurturing of innovative human resource; through the interaction between leading mathematicians and core research groups in pure mathematics, applied mathematics, statistics, theoretical physics, applied physics, theoretical biology and other relating disciplines, TSIMF will provide a platform for exploring new directions, developing new methods, nurturing mathematical talents, and working to raise the level of mathematical research in China.

About Facilities



Registration

Conference booklets, room keys and name badges for all participants will be distributed at the front desk. Please take good care of your name badge. It is also your meal card and entrance ticket for all events.



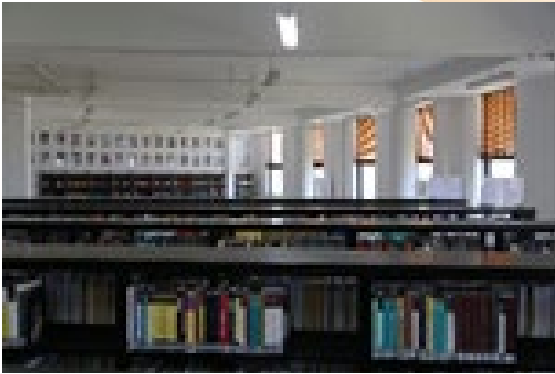
Guest Room

All the rooms are equipped with: free Wi-Fi (Password:tsimf123), TV, air conditioning and other utilities.

Family rooms are also equipped with kitchen and refrigerator.



Library



Opening Hours: 09:00am-22:00pm

TSIMF library is available during the conference and can be accessed by using your room card. There is no need to sign out books but we ask that you kindly return any borrowed books to the book cart in library before your departure.



In order to give readers a better understanding of the contributions made by the Fields Medalists, the library of Tsinghua Sanya International Mathematics Forum (TSIMF) instituted the Special Collection of Fields Medalists as permanent collection of the library to serve the mathematical researchers and readers.

So far, there are 271 books from 49 authors in the Special Collection of Fields Medalists of TSIMF library. They are on display in room A220. The participants are welcome to visit.



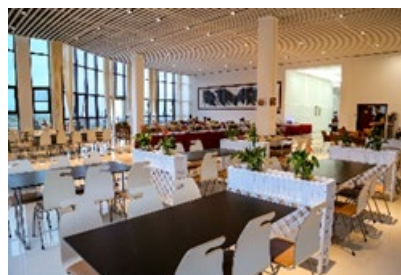
Restaurant

All the meals are provided in the restaurant (Building B1) according to the time schedule.

Breakfast 07:30-08:30

Lunch 12:00-13:30

Dinner 17:30-19:00



Laundry

Opening Hours: 24 hours

The self-service laundry room is located in the Building(B1).



Gym

Opening Hours: 24 hours

The gym is located in the Building 1 (B1), opposite to the reception hall. The gym provides various fitness equipment, as well as pool tables, tennis tables etc.



Playground

Playground is located on the east of the central gate. There you can play basketball, tennis and badminton. Meanwhile, you can borrow table tennis, basketball, tennis balls and badminton at the reception desk.

Swimming Pool

Please enter the pool during the open hours, swimming attire and swim caps are required, if you feel unwell while swimming, please stop swimming immediately and get out of the pool. The depth of the pool is 1.2M-1.8M.

Opening Hours: 13:00-14:00 18:00-21:00



Free Shuttle Bus Service at TSIMF

We provide free shuttle bus for participants and you are always welcome to take our shuttle bus, all you need to do is wave your hands to stop the bus.

Destinations: Conference Building, Reception Room, Restaurant, Swimming Pool, Hotel etc.



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