反问题的智能计算和数据科学

Intelligent Computing for Inverse Problems and Data Science

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Location

Venue: Room A-103, TSIMF

Organizers

Hongjie Li(李宏杰), Tsinghua University
Tiexiang Li(李铁香), Southeast University
Wenwei Lin(林文伟), Nanjing Center for Applied Mathematics
Jijun Liu(刘继军), Southeast University
Lingyun Qiu(邱凌云), Tsinghua University
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Titles and Abstracts

人工智能与数学---基于模型和数据的信息提取的理论与算法

Jin Cheng(程晋)
Fudan University/ Shanghai Key Laboratory for Contemporary Applied Mathematics
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人工智能与数学之间存在着紧密而不可分割的联系。这种联系不仅体现在数学为人工智能提供了坚实的基础和工具，也体现在人工智能对数学领域的发展和变革起到了重要的推动作用。在本报告中，我们将阐述：数学是人工智能的基础。许多人工智能算法和模型都建立在数学原理和方法的基础上。特别地，我们重点说明：基于模型和数据的信息提取技术为人工智能的发展提供了坚实的基础。现代数学研究的一个重要的动机是走向实际应用，解决应用中的难点和关键问题。 数学中的逆向思维、反问题与人工智能的创新紧密联系在一起。

Unique identification of the fractional order for a fractional SAIR epidemic model based on L-ADM asymptotic solution

Gongsheng Li(李功胜)
Shandong University of Technology
山东理工大学

This talk deals with an inverse order problem for a time-fractional nonlinear SAIR epidemic model by a single measurement. An asymptotic solution to the fractional SAIR model is derived by the Laplace Adomian decomposition method (L-ADM), and the inverse problem is transformed to a nonlinear equation and its uniqueness is proved under suitable conditions. Numerical inversions are presented by using an improved differential evolution algorithm.

Mathematical theory and applications for multi-layered medium

Youjun Deng(邓又军)
Central South University
中南大学

In this talk, we shall present some recent work on multi-layered medium. We shall build some elementary mathematical framework for multi-layered medium in electro-static, elasto-static and coupled systems. Based on the framework, we shall show some asymptotic results for multi-layered medium, which is a special case of inhomogeneous medium. We shall also show how to

design multi-layer structures of metamaterials, which may greatly increase the resonance modes, by using the algebraic framework we designed.

The effective construction on elastic metamaterials

Hongjie Li(李宏杰)
Tsinghua University
清华大学

In this talk, different kinds of structures are explored to effectively achieve the negative elastic metamaterials. First, the structure involving the bubbles embedded within the soft elastic materials will be discussed, offering an effective means to achieve the negative bulk modulus. Then the configuration incorporating hard inclusions embedded in the soft elastic matrix shall be investigated, enabling the attainment of negative values in the mass density.

Non-negative Tucker decomposition with graph regularization and smooth constraint for clustering

Linzhang Lu(卢琳璋)
Xiamen University/Guizhou Normal University
厦门大学/贵州师范大学

Non-negative Tucker decomposition (NTD) and its graph regularized extensions are the most popular techniques for representing high-dimensional non-negative data, which are typically found in a low-dimensional sub-manifold of ambient space, from a geometric perspective. Therefore, the performance of the graph-based NTD methods relies heavily on the low-dimensional representation of the original data. However, most existing approaches treat the last factor matrix in NTD as a low-dimensional representation of the original data. This treatment leads to the loss of the original data's multi-linear structure in the low-dimensional subspace. To remedy this defect, we propose a novel graph regularized *Lp* smooth NTD (GSNTD) method for highdimensional data representation by incorporating graph regularization and an L_p smoothing constraint into NTD. The new graph regularization term constructed by the product of the core tensor and the last factor matrix in NTD, and it is used to uncover hidden semantics while maintaining the intrinsic multi-linear geometric structure of the data. The addition of the L_p smoothing constraint to NTD may produce a more accurate and smoother solution to the optimization problem. The update rules and the convergence of the GSNTD method are proposed. In addition, a randomized variant of the GSNTD algorithm based on fiber sampling is proposed. Finally, the experimental results on four standard image databases show that the proposed method and its randomized variant have better performance than some other state-of-the-art graph-based regularization methods for image clustering.

A New Cross-Space Total Variation Regularization Model for Color Image Restoration with Quaternion Blur Operator

Zhigang Jia(贾志刚)
Jiangsu Normal University
江苏师范大学

The cross-channel deblurring problem in color image processing is difficult to solve due to the complex coupling and structural blurring of color pixels. Until now, there are few efficient algorithms that can reduce color infection in deblurring process. To solve this challenging problem, we present a novel cross-space total variation (CSTV) regularization model for color image deblurring by introducing a quaternion blur operator and a cross-color space regularization functional. The existence and uniqueness of the solution is proved and a new L-curve method is proposed to find a sweet balance of regularization functionals on different color spaces. The Euler-Lagrange equation is derived to show that CSTV has taken into account the coupling of all color channels and the local smoothing within each color channel. A quaternion operator splitting method is firstly proposed to enhance the ability of color infection reduction of the CSTV regularization model. This strategy also applies to the well-known color deblurring models. Numerical experiments on color image databases illustrate the efficiency and manoeuvrability of the new model and algorithms. The color images restored by them successfully maintain the color and spatial information and are of higher quality in terms of PSNR, SSIM, MSE and CIEde2000 than the restorations of the-state-of-the-art methods.

Mechanism Modeling and Numerical Computation for TCC/TPC Parameter Optimal Identification

Dinghua Xu(徐定华)
Shanghai University of Finance and Economics
上海财经大学

The talk presents the survey on the mechanism modeling and numerical computation for the thermal comfort clothing(TCC) and thermal protective clothing(TPC) design. The core mathematical technology comes from the mathematical description of temperature/moisture distribution in the human body-clothing-environment system with the aim of achieving the thermal comfort performance and thermal safety performance. Some models and numerical algorithms are presented to arrive at the goals. A successful application case of a special apparel in Bejing Winter Olympic Game 2022 is introduced.

Stability for Inverse Potential and Random Source Problems in Elastic Waves

Xiang Xu(徐翔)
Zhejiang University
浙江大学

In this talk, an inverse boundary value problem to recover a potential for elastic scattering equations from Dirichlet to Neumann map will be discussed. For an isotropic potential, an increasing stability result is derived, which consists of two parts: a Holder type part and a logarithmic type part vanishing when the frequency grows. For an anisotropic potential, a similar increasing stability of the linearized inverse problem is derived, by constructing different pairs of both real and complex exponential solutions, which give estimations of the Fourier modes of each entry in the potential matrix. Furthermore, a reconstruction algorithm is proposed aiming at the recovery of the Fourier modes of the potential function and is verified the efficiency by several numerical examples.

Inverse scattering of multiple particles based on the time reversal model

Jun Lai(赖俊)
Zhejiang University
浙江大学

This talk is concerned with the so-called DORT method that uses the eigenfunctions of time-reversal operator to determine the locations of small scatterers. We give a rigorous mathematical justification for the DORT method under different boundary conditions for elastic waves. Extension to inverse scattering in layered medium is also given. Utilizing the imaging result as an initial guess, a Bayesian inversion scheme is proposed to reconstruct the shape of multiple buried extended scatterers more accurately. The method is efficient, accurate and does not need a good initial guess.

Convergence of some numerical methods for parabolic inverse Robin problems

Daijun Jiang(蒋代军)
Central China Normal University
华中师范大学

We study in this talk some numerical methods for solving the highly nonlinear and ill-posed inverse problem of identifying the Robin coefficients in parabolic systems. We first apply the Levenberg-Marquardt method (LMM) to transform the Tikhonov regularized nonlinear non-convex minimizations into convex minimizations. And the quadratic convergence of the LMM is rigorously established for the nonlinear parabolic inverse problems for the first time, under a simple novel adaptive strategy for selecting regularization parameters during the LM iteration. Then the domain

decomposition methods (DDMs) are used to solve the convex minimizations. The methods are completely local and the local minimizers have explicit expressions within the subdomains. Numerical experiments are presented to show the accuracy and efficiency of the methods, in particular, the convergence seems nearly optimal in the sense that the iteration number of the methods is independent on the mesh size.

Some uniqueness results of inverse problems for nonlinear coupled PDEs

Guanghui Zheng(郑光辉)
Hunan University
湖南大学

Nonlinear coupled PDEs have widespread applications in materials science, fluid mechanics, electromagnetic wave propagation, ecology, biomedical engineering, game theory, and other fields. However, the inherent ill-posedness, nonlinear coupling, and special constraints (such as non-negativity or normalization constraints) of the solutions for the associated inverse problems make them particularly challenging. In this talk, I will present some recent results on the uniqueness of inverse problems involving nonlinear coupled PDEs. The uniqueness of the solutions to these inverse problems is a critical issue, as the nonlinearity and coupling introduce significant difficulties. I will discuss our latest findings on the uniqueness properties for these types of inverse problems across diverse application domains.

Cyclic stationary probability distribution of second order Markov chains and its applications

Dongdong Liu(刘冬冬)
Guangdong University of Technology
广东工业大学

In this talk, we define a cyclic stationary probability distribution equation for a second order Markov chain process in case that all states are independent each other, which improves the existing equation. There are two applications for the new model. First, the proposed model can be seen as a rank-3 approximation of a second order Markov chain with non-independent states. Secondly, unlike the previous tensor model, if the fixed point algorithm for solving the new model is convergent, the second order Markov chain process in the independent state converges. Furthermore, we investigate properties of the solutions for the proposed stationary equation.

Medical Imaging: Model-Driven VS Data-Driven

Dong Wang(王冬)
Nanjing Center for Applied Mathematics
南京应用数学中心

Medical imaging, such as CT, MRI, and PET, have been widely used in clinical diagnosis and treatment. In clinical settings, medical imaging usually suffers from long scanning time and large radiation dose. Therefore, fast, low-dose, and high-precision imaging is of great concern in medical imaging. In this talk, I will briefly review the common methods used in medical imaging and present our latest progress in fast dynamic MR imaging and low-dose PET imaging.

On a geometric principle in wave scattering and its applications

Hongyu Liu(刘宏宇)
Hong Kong City University
香港城市大学

In this talk, I shall discuss a geometric principle to bridge the micro and macro scales in wave scattering theory as well as its applications in super-resolution imaging and invisibility cloaking.

Convergence Analysis of Nonlinear Kaczmarz Method for Systems of Nonlinear Equations with Component-wise Convex Mapping

Chong Chen(陈冲)
Academy of Mathematics and Systems Science, CAS
中科院数学与系统科学研究院

Motivated by a class of nonlinear imaging inverse problems, for instance, multispectral computed tomography (MSCT), we study the convergence theory of the nonlinear Kaczmarz method (NKM) for solving the system of nonlinear equations with component-wise convex mapping, namely, the function corresponding to each equation being convex. However, such kind of nonlinear mapping may not satisfy the commonly used component-wise tangential cone condition (TCC). For this purpose, we propose a novel condition named relative gradient discrepancy condition (RGDC), and make use of it to prove the convergence and even the convergence rate of the NKM with several general index selection strategies, where these strategies include cyclic strategy and maximum residual strategy. Particularly, we investigate the application of the NKM for solving nonlinear systems in MSCT image reconstruction. We prove that the nonlinear mapping in this context fulfills the proposed RGDC rather than the component-wise TCC, and provide a global convergence of the NKM based on the previously obtained results. Numerical experiments further illustrate the

numerical convergence of the NKM for MSCT image reconstruction.

On the Early Stopping of Untrained Convolutional Neural Networks

Bangti Jin(金邦梯)
The Chinese University of Hong Kong
香港中文大学

In recent years, new regularization methods based on (deep) neural networks have shown very promising empirical performance for the numerical solution of ill-posed problems. Due to the nonlinearity of neural networks, these methods often lack satisfactory theoretical justification. In this talk, we rigorously discuss the convergence of one unsupervised approach utilizing untrained convolutional neural networks to represent solutions to linear ill-posed problems. The regularization property of the approach relies solely on the architecture of the neural network instead. We shall show that the classical discrepancy principle is an adequate method for early stopping of two-layer untrained convolutional neural networks learned by gradient descent, and furthermore, it yields an approximation with minimax optimal convergence rates. Numerical results are also presented to illustrate the theoretical findings.

Traceability of Water Pollution: An Inversion Scheme Via Dynamic Complex Geometrical Optics Solutions

Hui Yu(蔚辉)
Xiangtan University
湘潭大学

We investigate the identification of the time-dependent source term in the diffusion equation using boundary measurements. This facilitates tracing back the origins of environmental pollutants. Employing the concept of dynamic complex geometrical optics (CGO) solutions, a variational formulation of the inverse source problem is analyzed, leading to a proof of uniqueness result. Our proposed two-step reconstruction algorithm first determines the point source locations and subsequently reconstructs the Fourier components of the emission concentration functions. Numerical experiments on simulated data are conducted. The results demonstrate that the proposed two-step reconstruction algorithm can reliably reconstruct multiple point sources and accurately reconstruct the emission concentration functions. Additionally, by partitioning the algorithm into online and offline computations, and concentrating computational demand offline, real-time pollutant traceability becomes feasible. This method, applicable in various fields - especially those related to water pollution, can identify the source of a contaminant in the environment, thus serving as a valuable tool in environmental protection.

保度计算几何及其在医学图像处理中的应用

Tiexiang Li(李铁香)
Shing-Tung Yau Center of Southeast University/ Nanjing Center for Applied Mathematics
东南大学丘成桐中心/南京应用数学中心

计算几何、人工智能与医学影像的结合在医学领域中具有重要的意义。计算几何技术可以为医学影像提供精确的几何形状分析和建模能力，从而帮助医生更好地理解和诊断疾病。人工智能技术则可以通过机器学习和深度学习算法，自动识别和分析医学影像中的病变和异常情况，提高诊断的准确性和效率。这种结合为医学影像提供了更多的量化和定量化手段，为临床决策和患者护理提供了有力支持。此外，计算几何和人工智能的进步也推动了医学影像的图像重建、配准和分割等方面的技术创新。我们基于保度计算几何算法与人工智能技术结合用于处理 MRI 脑肿瘤影像分割与肿瘤类型分类取得了高精度效果，有希望在医疗诊断和治疗中发挥重要作用。

Finiteness of nonscattering wavenumbers

Jingni Xiao(肖晶妮)
Drexel University
德雷塞尔大学

We consider the non-scattering phenomenon of penetrable inhomogeneous media. For star-shaped domain admitting certain admissible conditions, we show that there are at most finitely many positive wave numbers at which Herglotz wave with a fixed density is nonscattering. This is based on joint work with Michael Vogelius.

Improved preconditioned Unsupervised K-means clustering algorithms

Xiaofei Peng(彭小飞)
South China Normal University
华南师范大学

The Unsupervised K-means clustering (UKM) algorithm has attracted the attention of many researchers because it can automatically identify the number of clusters without requiring any parameter selection. However, it may produce poor clustering results on datasets with Gaussian mix tures. In this paper, we consider the two-step UKM algorithm, where the truncated UKM algorithm is first used as a preconditioning strategy. To further enhance the algorithm's performance, we introduce a circular modification technique. In addition, we optimize the cluster results by leveraging the Bayesian Information Criterion (BIC). The experimental results reveal that the

proposed algorithms have a higher clustering accuracy than the UKM when applied to Gaussian mixture datasets.

Metal Artifact Removal via adaptive and learned weights

Huibin Chang(常慧宾)
Tianjin Normal University
天津师范大学数学科学学院

The existence of metal artifacts severely affects the quality of CT reconstruction, posing challenges to subsequent analysis and diagnosis. This report will share our team's newly proposed adaptive and learned weight-based metal artifact correction and reconstruction methods, and demonstrate the effectiveness of the method with a large amount of simulation and actual data.

Simultaneous determination of the order and a coefficient in a fractional diffusion-wave equation

Ting Wei(魏婷)
Lanzhou University
兰州大学

This paper recovers the order of fractional derivative and a time-dependent potential coefficient in a time-fractional diffusion wave equation by an integral condition or one single point measurement on the boundary. The Lipschitz continuity of the forward operators from the unknown order and coefficient to the given data are achieved in terms of the integral equation held by the solution of the direct problem. We also obtain the uniqueness for the considered inverse problems in terms of somewhat general conditions to the given functions. Moreover, we propose a Tikhonov-type regularization method and prove the existence of the regularized solution and its convergence to the exact solution under a suitable regularization parameter choice. Then we use a linearized iteration algorithm to recover numerically the order and time-dependent potential coefficient simultaneously. Three numerical examples for one- and two-dimensional cases are provided to display the efficient of the proposed method.

Curvature regularization and its applications in Non-Line-of-Sight Imaging

Yuping Duan(段玉萍)
Beijing Normal University
北京师范大学

The geometric high-order regularization methods such as Euler's elatica, mean curvature, and Gaussian curvature, have been intensively studied during the last decades due to their abilities in preserving geometric properties including image edges, corners, and image contrast. However, the dilemma between restoration quality and computational efficiency is an essential roadblock for high-order methods. We propose a new variational model for image reconstruction by minimizing the L1 morn of the Weingarten map of image surface. We analytically prove that the Weingarten map minimization model can not only keep the greyscale intensity contrasts of images, but also preserve edges and corners of objects. In what follows, we derive a hybrid nonlinear first and second order regularization from the Weingarten map, and present an efficient ADMM-based algorithm by regarding the nonlinear weights as known. Besides, we also extend the application of curvature energies for non-line-of-sight (NLOS) imaging. Both traditional and learnable curvature regularization model are developed for NLOS with under-scanning measurements.

A mechanism learning based method for data filling

Yu Chen(陈瑜)
Shanghai University of Finance and Economics
上海财经大学

This talk is concerned with an inpainting method based on mechanism learning, from the perspective of inverse problems. The underlying data mechanism, characterized by linear differential equations, is identified from data on the known area and then exploited to infer that on the missing part, which gives interpretability. Attention is paid to incorporation of historical or prior information as higher order mechanism. Numerical examples show effectiveness, robustness and flexibility of the method and it performs well over mechanism/scientific data. This is a joint work with Prof. Jin Cheng.