

几何、量子场论和非线性分析的相互作用 The interaction between geometry, quantum field theory and nonlinear analysis

Date

2024-03-18 ~ 2024-03-23

Location

Venue: Room A-103, TSIMF

Organizer

Jürgen JOST, Max Planck Institute for Mathematics in the Sciences

CHEN Qun(陈群), Wuhan University

ZUO Kang(左康), Wuhan University

Proposal

The interaction between geometry (Riemannian, algebraic, Kaehler, metric,...), quantum field theory and analysis (elliptic and parabolic PDEs, calculus of variations) has brought spectacular advances and generated deep insight in all those disciplines.

Minima of variational integrals may provide optimal solutions to geometric problems, and parabolic PDEs may deform initial geometrical structures into optimal ones, thereby solving important geometric problems. Functionals from QFT contain rich structures that can be exploited for the construction of subtle geometric invariants. In turn, such problems typically lead to very difficult analytical challenges. The resulting PDEs are not only highly non-linear, but from a variational perspective usually are not contained in the range of the Palais-Smale condition, and therefore, standard methods usually break down. This challenge gave an important impetus to the theory of non-linear PDEs. One line of research exploited convexity properties, typically arising from non-positive curvature, another one depended on an extremely careful study of the formation of singularities, which in turn had to use geometric features or algebraic properties from QFT.

During this workshop, we want to bring people together to explore various current research questions in this field, including

- Analytic methods for studying moduli spaces in algebraic geometry
- A general mathematical theory of the action functionals of QFT and the resulting challenges for PDE theory to construct minimizers or other critical points and to understand their regularity properties
- Relations between QFT functionals and geometric constructions, like Kapustin-Witten and Higgs fields
- Bernstein and Dirichlet problems for minimal submanifolds of Euclidean spaces and spheres
- The role of PDEs in metric space geometry
- The geometry of positive sectional curvature
- The approximation of geometric objects by discrete ones

Schedule

Time&Date	Monday (March 18)	Tuesday (March 19)	Wednesday (March 20)	Thursday (March 21)	Friday (March 22)
7:30-8:30	<i>Breakfast (60 minutes)</i>				
<i>Chair</i>					
8:50-9:00	Opening (10minutes)				
9:00-9:45	Wilderich Tuschmann	XIA Chao	FAN Huijun	ZHANG Huichun	LIU Shiping
10:00-10:45	WANG Guofang	DING Qi	Enno Keßler	HUA Bobo	ZHOU Chunqin
11:00-11:30	<i>Coffee Break (within 30 minutes)</i>				
<i>Chair</i>					
11:30-12:15	ZHANG Xi	Renan Assimos	WU Ruijun	JIANG Shuhan	AI Wanjun
12:25-13:55	<i>Lunch (90 minutes)</i>				
<i>Chair</i>					
15:30-16:15	ZHU Miaomiao	QIU Hongbing	Free Discussion 13:30-17:00	ZHAO Liang	GUI Yaoting
16:30-17:00	<i>Coffee Break (within 30 minutes)</i>			<i>Coffee Break (within 30 minutes)</i>	
<i>Chair</i>					
17:00-17:45	SUN Linlin	LIU Lei		ZHU Jingyong	
18:10-19:30	<i>Dinner</i>		Banquet 18:00- 20:00	<i>Dinner</i>	

Titles and Abstracts

TOPOLOGICAL PROPERTIES AND COMPACTIFICATIONS OF MODULI SPACES OF RIEMANNIAN METRICS

Wilderich Tuschmann
Karlsruhe University

Consider a smooth manifold with a Riemannian metric satisfying some specific sort of geometric constraint like, for example, having positive scalar curvature, non-negative Ricci or negative sectional curvature, being Einstein, Kähler, Sasakian, etc.

A natural question to ponder is then what the 'space' of all such metrics does look like. Moreover, one can also study this question for the corresponding moduli spaces of metrics, i.e., quotients of the former by (suitable subgroups of) the diffeomorphism group of the manifold, acting by pulling back metrics.

These objects are customarily equipped with the topology of smooth convergence on compact subsets and the quotient topology, respectively. Their topological properties then provide the right means to measure 'how many' different metrics and geometries the given manifold actually does exhibit and since Teichmüller's and Weyl's early and seminal works in dimension two, the study of spaces of metrics and their moduli has been a topic of interest for differential geometers, global and geometric analysts, and topologists alike.

In my talk, after providing a short introduction to the subject with a focus on metrics with nonnegative curvature bounds, I will present a non-traditional metric approach to the study of moduli spaces via Gromov-Hausdorff convergence techniques which, in particular, also allows for the study of natural compactifications. Moreover, I will describe the moduli spaces of flat and certain Ricci flat metrics on closed manifolds in a new way via symmetric spaces and, given time permits, also discuss further results and applications arising from these descriptions.

A higher order Sobolev inequality

WANG, GUOFANG
Freiburg University

In the talk I will propose a new type optimal Sobolev inequality and prove a special case as evidence. I will talk about possible approaches to prove the general case and difficulties.

Mean curvature positivity and rational connectedness

ZHANG,XI

Nanjing University of Science and Technology

In this talk, we consider mean curvature positivity of holomorphic vector bundle. We will show that the mean curvature positivity is equivalent to the HN-positivity. As its applications, We establish the correspondence between rational connectedness in algebraic geometry and mean curvature positivity in differential geometry. We also introduce our recent work on the rational connectedness of compact Kähler manifolds which extends Yau's conjecture to the quasi-positive case. These works are joint with Chao Li, Shiyu Zhang and Chuanjing Zhang.

Qualitative behavior at the free boundary for harmonic map type problems

ZHU,MIAOMIAO

Shanghai Jiao Tong University

In this talk, we shall present some recent progress on the qualitative behavior at the free boundary for some conformally invariant geometric variational Lagrangians.

Some results related to the Kazdan-Warner equations

SUN,LINLIN

Guangxi Normal University

The Kazdan-Warner equation on surface comes from the prescribed Gaussian curvature problem, and also appears in various contexts such as the abelian Chern-Simons-Higgs models. I shall talk about some results related to the Kazdan-Warner equations on surfaces or finite graphs, including the elliptic method and parabolic approach to the Kazdan-Warner equations on surfaces with sign-changed prescribing function, as well as the topological method to the existence of Kazdan-Warner equations on surfaces or finite graphs. This talk is based on the joint works with Prof. Li Jiayu, WANG Liuquan, YANG Yunyan and ZHU Jingyong.

A Willmore-type inequality for free boundary hypersurfaces in unbounded convex domains

XIA,CHAO

Xiamen University

We shall discuss a Willmore-type inequality for free boundary hypersurfaces in an unbounded convex domain, which says that the total curvature of the free boundary hypersurfaces is bounded below by the asymptotic volume ratio of the domain. This Willmore inequality leads to a relative isoperimetric inequality.

Minimal graphs over manifolds

DING,QI

Fudan University

We will talk about properties of minimal graphs over manifolds, including Liouville type theorems, Neumann-Poincare inequality, gradient estimates, asymptotic estimates, splitting theorems and so on.

On a new Omori-Yau maximum principle for harmonic maps

Renan Assimos

Hannover University

By introducing a concept generalising several convexity notions we obtain a new Omori-Yau maximum principle for harmonic maps defined on a stochastically complete manifold. Some of the applications of this new maximum principle include conformal harmonic maps, an adaptation of a conjecture of Calabi, harmonic immersions with certain energy bounds, wedge theorems for minimal submanifolds of \mathbb{R}^n and a walled halfspace theorem.

Rigidity of of ancient solutions to the mean curvature flow in higher codimension

QIU,HONGBING

Wuhan University

In this talk, we shall discuss ancient solutions to the mean curvature flows. By carrying out refined curvature estimates, we prove better rigidity theorems of complete noncompact ancient solutions

to the mean curvature flow in higher codimension under various Gauss image restriction. This is a joint work with professor Y. L. Xin.

Three circle lemma and its applications in some geometric variational problems

LIU,LEI

Central China Normal University

In this talk, we will first recall the three circle lemma for harmonic functions and then give its applications in the blow-up analysis of some geometric variational problems, such as harmonic maps, Dirac-harmonic maps and so on. Precisely, we will prove some energy identities in the blow-up process.

Geometry of Landau-Ginzburg Models

FAN,HUIJUN

Peking University

An LG model (M, f) is given by a noncompact complex manifold M and the holomorphic function f defined on it, which is an important model in string theory. This talk gives a survey of my work on the study of differential geometric structure of a LG model since 2011. We starts with some examples, gives the geometric and topological information contained by a LG model, and then consider the Schrodinger equations and their deformation theory of a family of LG models. The output is the Hodge variation structures, Gauss-Manin connections, index theory and torsion invariants.

Super Stable Maps and Super Gromov-Witten Invariants

Enno Keßler

Max Planck Institute for Mathematics in the Sciences

J-holomorphic curves or pseudoholomorphic curves are maps from Riemann surfaces to almost Kähler manifolds satisfying the Cauchy-Riemann equations. The moduli space of J-holomorphic curves has a natural compactification using stable maps. Moduli spaces of stable maps are of great interest because they allow to construct invariants of the target manifold and those invariants are deeply related to topological superstring theory.

In this talk, I want to report on a supergeometric generalization of J-holomorphic curves, stable maps and Gromov-Witten invariants where the domain is a super Riemann surface. Super Riemann surfaces have first appeared in superstring theory as generalizations of Riemann surfaces with an

additional anti-commutative dimension. Super J-holomorphic curves are solutions to a system of partial differential equations on the underlying Riemann surface coupling the Cauchy-Riemann equation with a Dirac equation for spinors. I will explain how to construct moduli spaces of super J-holomorphic curves and super stable maps in genus zero via super differential geometry and geometric analysis.

Motivated by the super moduli spaces I give an algebro-geometric proposal for super Gromov-Witten invariants satisfying generalized Kontsevich-Manin axioms.

Nodal sets of solutions to Dirac equations

WU,RUIJUN

Beijing Institute of Technology

Motivated from some geometric problems, we study the nodal sets of solutions to general Dirac equations. We obtain a dimension estimate via approximating the solutions by harmonic spinors. Further under some growth condition we show that the nodal sets are stratified, using an adapted version of the frequency function for spinors. These generalize the previous well-known results for smooth solutions.

This is a joint work with W.Borrelli.

Boundary regularity of harmonic maps from RCD spaces to CAT(0)-spaces

ZHANG,HUICHUN

Sun Yat-sen University

In this talk, we will introduce the boundary regularity for harmonic maps from RCD(K,N) spaces, a class of metric measure spaces with generalized Ricci curvature bounded from below, into non-positively curved space in the sense of Alexandrov spaces. This is a joint work with Xi-Ping Zhu.

Graphs with nonnegative Ollivier Ricci curvature

HUA,BOBO

Fudan University

We study the class of graphs with nonnegative Ricci curvature in the sense of Ollivier, Lin-Lu-Yau, et al. We prove the number of ends is at most two for an infinite graph with nonnegative Ricci curvature. Moreover, we prove that the grid graph, a discrete analog of the Euclidean space, is rigid in the following sense: any nontrivial local perturbation of a grid graph produces negative Ricci curvature somewhere. The key tools are harmonic functions of linear growth. These are joint works

with Florentin Muench and Haohang Zhang.

Supersymmetric Generalized Seiberg-Witten Theory in Dimension 4

JIANG,SHUHAN

Max Planck Institute for Mathematics in the Sciences

In this talk, I will introduce a supersymmetric extension of the 4D generalized Seiberg-Witten functional, thereby establishing it as a cohomological field theory. I will discuss its connections to the 4D Super Yang-Mills theory. If time allows, I will also touch upon its perturbative quantization.

Sobolev Spaces on Graphs and Their Applications

ZHAO,LIANG

Beijing Normal University

We consider Sobolev spaces and their several basic properties on locally finite graphs. Unlike the Euclidean case, since there is no exact concept of dimension on graphs, we introduce a space composed of vector-valued functions with variable dimensions to address this issue. As fundamental analytical tools, these results would be extremely useful for partial differential equations on graphs. We will discuss their applications in several nonlinear equations or systems on graphs.

The report is based on collaborative works with Han X.L., Shao M.Q., Yang Y.Y, etc.

Existence and uniqueness of Dirac-harmonic maps

ZHU,JINGYONG

Sichuan University

As a commutative version of the supersymmetric nonlinear sigma model, Dirac-harmonic maps from Riemann surfaces were introduced almost twenty years ago. They are critical points of an unbounded conformally invariant functional involving two fields. As solutions to a coupled nonlinear elliptic system, the existence theory of Dirac-harmonic maps has already received much attention. In this talk, we present some results on the existence and the uniqueness of Dirac-harmonic maps based on the joint works with Professor Jürgen Jost.

Isoperimetry and extremal eigenvalues of Cayley graphs

LIU,SHIPING

University of Science and Technology of China

We will discuss certain symmetry of the spectra of the normalized adjacency matrices of Cayley graphs in terms of their vertex/edge isoperimetry. This improves upon recent obtained estimates aiming at a quantitative version of an observation due to Breuillard, Green, Guralnick and Tao. We use spectral theory of signed graphs as a tool. This is based on joint works with Chunyang Hu and with Wenbo Li.

Anisotropic Moser-Trudinger inequalities and the related inequalities

ZHOU,CHUNQIN

Shanghai Jiao Tong University

In this talk, I mainly introduce some ideas that be used to extend the classical Moser-Trudinger inequality to its anisotropic type. Similar arguments can be applied for establishing the anisotropic weighted Moser-Trudinger inequalities and the anisotropic Adams' inequality. These are joint works with Changliang Zhou, Tao Zhang and Tingzhi Cheng.

Some Variational Results of the Generalized Seiberg-Witten Functional

AI,WANJUN

Southwest University

In this presentation, I will delve into the exploration of a generalized Seiberg–Witten functional, which unifies the classical Seiberg–Witten and Kapustin–Witten functionals. These gauge functionals, derived from quantum field theory, have demonstrated profound applications in various mathematical contexts. The talk will provide insights into the existence and regularity of critical points within this generalized framework. This work is a collaboration with Shuhan Jiang and Jürgen Jost.

**LIPSCHITZ REGULARITY OF HARMONIC MAPS FROM HEISENBERG GROUP INTO
CAT(0) SPACE**

GUI, YAOTING

Beijing International Center for Mathematical Research (BICMR)

We prove the local Lipschitz continuity of energy minimizing harmonic maps between singular spaces, more specifically from the n -dimensional Heisenberg group into CAT(0) spaces. This is a jointwork with Renan Assiros and Jiirgen Jost.

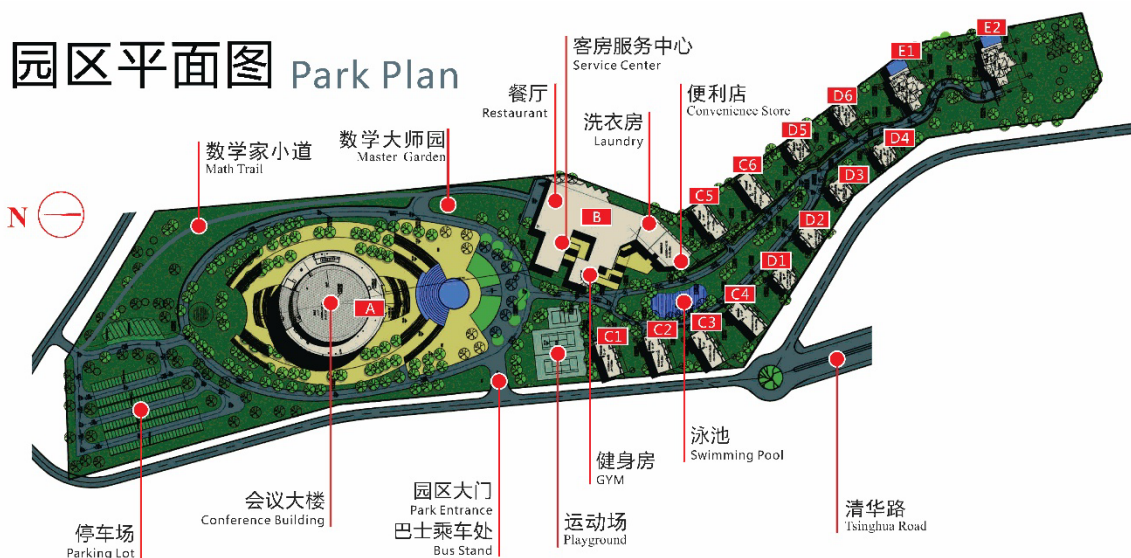


Welcome to TSIMF

The facilities of TSIMF are built on a 23-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 29,000 square meter that includes state-of-the-art conference facilities (over 10,000 square meter) to hold many international workshops simultaneously, two reading rooms of library, a guest house (over 10,000 square meter) and the associated catering facilities, a large swimming pool, gym and sports court and other recreational facilities.

Management Center of Tsinghua Sanya International Forum is responsible for the construction, operation, management and service of TSIMF. The mission of TSIMF is to become a base for scientific innovations, and for nurturing of innovative human resource; through the interaction between leading mathematicians and core research groups in pure mathematics, applied mathematics, statistics, theoretical physics, applied physics, theoretical biology and other relating disciplines, TSIMF will provide a platform for exploring new directions, developing new methods, nurturing mathematical talents, and working to raise the level of mathematical research in China.

About Facilities



Registration

Conference booklets, room keys and name badges for all participants will be distributed at the front desk. Please take good care of your name badge. It is also your meal card and entrance ticket for all events.

Guest Room



All the rooms are equipped with: free Wi-Fi (Password: [tsimf123](#)), TV, air conditioning and other utilities.

Family rooms are also equipped with kitchen and refrigerator.

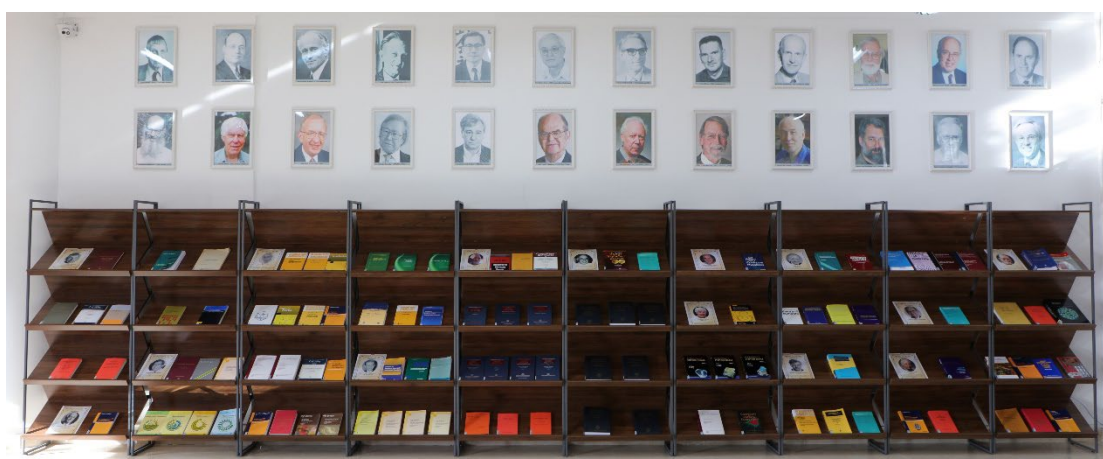


Library



Opening Hours: 09:00am-22:00pm

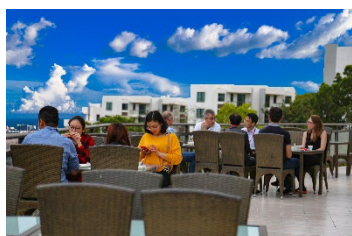
TSIMF library is available during the conference and can be accessed by using your room card. There is no need to sign out books but we ask that you kindly return any borrowed books to the book cart in library before your departure.



In order to give readers a better understanding of the contributions made by the Fields Medalists, the library of Tsinghua Sanya International Mathematics Forum (TSIMF) instituted the Special Collection of Fields Medalists as permanent collection of the library to serve the mathematical researchers and readers.

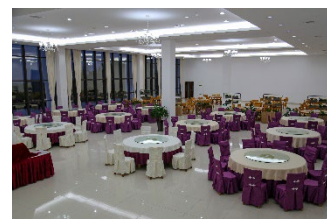
So far, there are 271 books from 49 authors in the Special Collection of Fields Medalists of TSIMF library. They are on display in room A220. The participants are welcome to visit.

Restaurant



Breakfast	07:30-08:30
Lunch	12:00-13:30
Dinner	17:30-19:00

All the meals are provided in the restaurant (Building B1) according to the time schedule.



Laundry

Opening Hours: 24 hours

The self-service laundry room is located in the Building(B1).



Gym

The gym is located in the Building 1 (B1), opposite to the reception hall. The gym provides various fitness equipment, as well as pool tables, tennis tables etc.

Playground



Playground is located on the east of the central gate. There you can play basketball, tennis and badminton. Meanwhile, you can borrow table tennis, basketball, tennis balls and badminton at the reception desk.

Swimming Pool

Please note that there are no lifeguards. We will not be responsible for any accidents or injuries. In case of any injury or any other emergency, please call the reception hall at +86-898-38882828.



Free Shuttle Bus Service at TSIMF

We provide free shuttle bus for participants and you are always welcome to take our shuttle bus, all you need to do is wave your hands to stop the bus.



Destinations: Conference Building, Reception Room, Restaurant, Swimming Pool, Hotel etc.



Contact Information of Administration Staff

Location of Conference Affairs Office: Room 104, Building A

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Email: yeli@tsimf.cn

*Reception duty hours: 7:00-23:00, chamber service please call: 0086-38882828
(exterior line) 80000 (internal line)

*Room maintainer night duty hours: 23:00-7:00, if you need maintenance services,
please call: 0086-38263909 (exterior line) 30162 (internal line)

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