# Workshop on Algebraic Representation Theory and Related Topics

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# Abstracts

**Susumu Ariki** (Osaka University)

*Tame blocks of Hecke algebras*

I explain how theories developed for studying Hecke algebras of classical type combined with results from the representation theory of finite dimensional algebras allowed us to determine Morita classes of tame blocks of Hecke algebras of classical type over an algebraically closed field of odd characteristic. This generalized an old result for blocks of type $A$, the proof of which used the Scopes' equivalence.

**Tristan Bozec** (Université Claude Bernard Lyon 1)

*From representations of quivers to bundles on curves*

There exists a well known and fruitful analogy between moduli spaces $M_Q$ of representations of quivers on one hand, and moduli spaces $Bun_X$ of vector bundles on curves on the other hand. For instance and for categorical reasons, associated Hall algebras behave similarly. In this talk I will describe an approach based on a curve analog of Kac polynomials, introduced by Schiffmann, and will explain how these are precisely related to the original ones - on the quiver side. It gives a formula for the number of irreducible components of a global analog of the nilpotent cone $\Lambda_X$, which is a Lagrangian subvariety of $T^*Bun_X$, and I will explain the combinatorics giving the description of these components, as well as the stable ones (with resect to the usual slope).

**Aslak Bakke Buan** (Norwegian University of Science and Technology )

*Wide subcategories and tau-exceptional sequences*

Based on joint work with Robert Marsh. We introduce the notion of (signed) tau-exceptional sequences for finite dimensional algebras, generalizing exceptional sequences for hereditary algebras. The sequences can in particular be seen as compositions of maps in a certain category, where the objects are wide subcategories of the module category.

**Xiao-Wu Chen** (University of Science and Technology of China)

*The lower extension groups and quotient categories*

I will review an old result of Keller-Vossieck on an explicit construction of a realization functor for a bounded $t$-structure in an algebraic triangulated category. Indeed, Keller-Vossieck's construction is more general. In analyzing their construction, we propose the following question: for a full additive subcategory $X$ in an additive category $A$, when the canonical functor from $K^b(A)/K^b(X)$ to $K^b(A/[X])$ is an equivalence. Here, $A/[X]$ is the additive quotient category. The answer is related to the lower extension groups in relative homological algebra. Moreover, the dg quotient category $A/X$ in the sense of Keller and Drinfeld appears naturally. This is joint with Xiaofa Chen in USTC.

**JiaRui Fei** (Shanghai Jiao Tong University)

*Combinatorics of $F$-polynomials*

We introduce the stabilization functors to study the combinatorial aspect of the $F$-polynomial of a representation of any finite-dimensional basic algebra. The $F$-polynomial of a quiver representation $M$ is the generating series of the topological Euler characteristic of the representation Grassmannian of $M$:

$$F_M(y) = \sum_{\gamma} \chi(\mathrm{Gr}_{\gamma}(M)) y^{\gamma}.$$

We characterize the vertices of their Newton polytopes. We give an explicit formula for the F-polynomial restricting to any face of its Newton polytope. For acyclic quivers, we give a complete description of all facets of the Newton polytope when the representation is general. We also prove that the support of F-polynomial is saturated for any rigid representation. We provide many examples and counterexamples, and pose several conjectures.

**Changjian Fu** (Sichuan University)

*On cluster algebras arising from weighted projective lines*

For each hereditary abelian category $\mathcal{H}$ (over an algebraic closed field) with tilting objects, one can construct a cluster algebra associated to $\mathcal{H}$. According to Happel's classification theorem, $\mathcal{H}$ is either derived equivalent to the module category of a finite dimensional hereditary algebra or the category of coherent sheaves of a weighted projective line.

When $\mathcal{H}$ is derived equivalent to the module category of a hereditary algebra, the associated cluster algebra is acyclic, which has been well investigated. In this talk, we will focus on certain properties of the cluster algebras associated to weighted projective lines.

**Jin Yun Guo** (Hunan Normal University)

*$\mathbb{Z}Q$ type constructions in higher representation theory*

Let $Q$ be an acyclic quiver, it is classical that certain truncations of the translation quiver $\mathbb{Z}Q$ appear in the Auslander-Reiten quiver of the path algebra $kQ$. We introduce the $n$-translation quiver $\mathbb{Z}|_{n-1}Q$ as a generalization of the $\mathbb{Z}Q$ construction in studying $n$-translation algebras. We find a class of algebras $\Gamma$ of global dimension $n$, called $(n-1)$-mesh algebras, for which the $\tau_n$-closure of its dual and $\tau_n^{-1}$-closure of it can be truncated from $\mathbb{Z}|_{n-1}Q$, as the preinjective and preprojective components do from $\mathbb{Z}Q$ for the hereditary algebra.

**Min Huang** (The University of Hong Kong)

*Quantum cluster algebras from unpunctured surfaces as non-commutative surface algebras*

Berenstein and Retakh recently introduced a class of non-commutative algebras with non-commutative Laurent phenomenon from marked surfaces. In this talk, we will realize quantum cluster algebras from unpunctured surfaces as quotient of the non-commutative surface algebras. As an application, a new expansion formula for quantum cluster algebras from unpunctured surfaces will be given. This is a joint work with Berenstein and Retakh.

**Akishi Ikeda** (Osaka University)

*Stability conditions on triangulated categories arising from marked bordered surfaces*

The notion of a stability condition was introduced by T.Bridgeland and he showed that the set of all stability conditions(called the space of stability conditions) becomes a complex manifold. Recently, there appear various results which identity the space of stability conditions on a triangulated arising from a marked bordered surface with a moduli space of quadratic differentials on a Riemann surface. In this talk, we review these type of results by Bridgeland-Smith, Haiden-Katzarkov-Kontsevich and my joint work with Yu Qiu.

**Osamu Iyama** (Nagoya University)

*Cohen-Macaulay DG modules and Calabi-Yau configurations, after H. Jin*

This talk is based on Haibo Jins work (arXiv:1812.03737). For a proper non-positive Gorenstein DG algebra A, we study the category CM A of Cohen-Macaulay DG A-modules. The category CM A forms a Frobenius extriangulated category in the sense of Nakaoka-Palu, and the stable category is canonically triangle equivalent to the singularity category of Buchweitz and Orlov. Moreover CM A admits almost split sequences and Auslander-Buchweitz approximations. When A is d-selfinjective (i.e. DA is isomorphic to A[-d]) and has only finitely many indecomposable CM DG modules, we describe the Auslander-Reiten quiver of CM A in terms of -(d+1)-Calabi-Yau configurations, studied independently by Coelho Simoes and Pauksztello. This is a generalization of Riedtmanns classification of the Auslander-Reiten quivers of representation-finite selfinjective algebras in 1980.

**Bernhard Keller** (Université Paris Diderot - Paris 7)

*Analytic potentials and the integration map*

Recall that a (complex) potential on a finite quiver is an infinite linear combination of cycles. It is analytic if the coefficients grow no faster than those of a geometric series. As shown by Toda, each semi-simple family of stable coherent sheaves on a projective CY 3-fold gives rise to a quiver with analytic potential whose stack of finite-dimensional representations is locally the stack of semi-stable sheaves filtered by the family. We prove that each quiver without loops and 2-cycles admits an analytic potential and that analyticity is preserved under mutations. Following Kontsevich-Soibelman, Joyce-Song and Nagao, for a quiver with analytic potential, we construct a Behrend-weighted integration map on the stack of finite-dimensional representations of a quiver with analytic potential. Conjecturally, this should allow to construct an automorphism of the upper cluster algebra associated with each torsion substack of the stack of finite-dimensional modules, in such a way that deformations of the substack do not modify the automorphism.

**Steffen Koenig** (Universität Stuttgart)

*Schur-Weyl duality, characteristic subcategories and homological dimensions*

Classical Schur-Weyl duality provides a close connection between group algebras of symmetric groups and Schur algebras, through a double centraliser property. What happens on the level of derived categories? Chuang and Rouquier have constructed derived equivalences between blocks of symmetric groups, and very similar equivalences between blocks of Schur algebras. Conversely, derived equivalences between blocks of Schur algebras always restrict to derived equivalences between blocks of symmetric groups, since their derived categories can be identified with characteristic subcategories. Moreover, the derived equivalences between blocks of Schur algebras preserve global and dominant dimensions.

As an application, global and dominant dimensions of blocks of Schur algebras can be computed. It turns out that global dimension is a complete invariant of derived equivalence classes in this case.

The general results on characteristic subcategories and invariance of some homological dimensions work for a large class of algebras including Morita algebras and thus also gendo-symmetric algebras.

(Joint work with Ming Fang and Wei Hu.)

**Julian Külshammer** (University of Uppsala)

*A functorial approach to monomorphism categories for species*

The monomorphism category of the module category of an algebra A was studied by Ringel and Schmidmeier around 2006. It forms a resolving subcategory of the module category of the triangular matrix ring of A which contains the Gorenstein projective modules. In this talk we propose two functorial approaches to monomorphism categories. One of them uses Auslander's approach of coherent functors, seeing the monomorphism category as restrictions of left exact functors from the module category of A to vector spaces. The other one uses Kvamme's theory of adjunctions with Nakayama functors and realises the monomorphism category as the category of (relative) Gorenstein projectives with respect to the Eilenberg-Moore adjunction with respect to a particular comonad. As an application, we can generalise the results of Ringel and Schmidmeier to the category of representations of a (generalised) species. This is based on joint work with Nan Gao, Sondre Kvamme, and Chrysostomos Psaroudakis.

**Yu Liu** (Southwest Jiaotong University)

*Abelian categories arising from cluster tilting subcategories*

In this talk, we consider a kind of ideal quotient of an extriangulated category such that the ideal is the kernel of a functor from this extriangulated category to an abelian category. We study a condition when the functor is dense and full, in another word, the ideal quotient becomes abelian. Moreover, a new equivalent characterization of cluster tilting subcategories is given by applying homological methods according to this functor. As an application, we show that in a connected 2-Calabi-Yau triangulated category $\mathcal{B}$, a functorially finite, extension closed subcategory $\mathcal{T}$ of $\mathcal{B}$ is cluster tilting if and only if $\mathcal{B}/\mathcal{T}$ is an abelian category.

**Amnon Neeman** (Australian National University)

*Approximable triangulated categories*

Approximable triangulated categories are compactly generated triangulated categories that are assumed to satisfy a couple of extra assumptions - as far as I know no one has looked at this before, in spirit the idea is that these triangulated categories carry a metric, and the objects of the category admit good approximations by simpler objects with respect to the metric.

In the talk the main results will be that (1) there are lots of examples, and (2) knowing that a triangulated category is approximable allows one to prove powerful consequences.

To spell out (2) a little more: time permitting we'll indicate how this structure was applied to (a) prove strong generation results, and (b) greatly simplify the proof of Serre's GAGA theorem. Since the theory is in its infancy it is to be hoped that more applications will come.

**Matthew Pressland** (University of Stuttgart)

*Calabi-Yau singularity categories*

Starting from the data of a quiver with potential (Q,W), I will explain how to construct an algebra B which is Gorenstein with 2-Calabi-Yau singularity category whenever it is Noetherian. Conjecturally, this singularity category is equivalent to Amiot's cluster category for (Q,W), and both Noetherianity of B and the equivalence are proved to hold when Q is acyclic. The construction appears to be related to the Ginzburg dg-algebra of (Q,W), and I will discuss both precise and conjectural connections to this dg-algebra.

**Fan Qin** (Shanghai Jiao Tong University)

*Bases for upper cluster algebras and tropical points*

It is known that many (upper) cluster algebras possess very different good bases which are parametrized by the tropical points of Langlands dual cluster varieties. For any given injective reachable upper cluster algebra, we describe all of its bases parametrized by the tropical points. In addition, we obtain the existence of the generic bases for such upper cluster algebras. Our results apply to many cluster algebras arising from representation theory, including quantized enveloping algebras, quantum affine algebras, double Bruthat cells, etc.

**Shiquan Ruan** (Xiamen University)

*Recollements and Ladders for weighted projective lines*

In this talk, we will introduce a method to construct recollements and ladders for exceptional curves by using reduction/insertion functors due to p-cycle construction. As applications to weighted projective lines, we classify recollements for the category of coherent sheaves over a weighted projective line, and give an explicit description of ladders in two different levels: the bounded derived category of coherent sheaves and the stable category of vector bundles.

**Kyoji Saito** (Kyoto University)

*Elliptic Artin group*

Artin groups (introduced 1972, called also generalized braid groups) appear as fundamental group of regular orbit spaces of Weyl groups, and are presented by braid relations associated with the Dynkin diagram. They play basic roles both in geometry and representation theory. According to the generalization of the classical root systems to elliptic root systems, we introduce elliptic Artin groups. They first appear as the fundamental group of regular orbit space of elliptic Weyl group action, but are presented by generalized Artin braid relations defined on elliptic diagrams. One crucial difference of Elliptic Artin groups from the classical Artin groups is that they admit an action of the central extensions of elliptic modular groups: $\Gamma_0(1), \Gamma_0(2)$ or $\Gamma_0(3)$. This fact is based on the fact that the rank 2 radical of an elliptic root system is identified with the homology lattice of elliptic curves of the Weierstrass, Legendre and Hesse family, respectively.

**Sibylle Schroll** (University of Leicester)

*A complete derived invariant for gentle algebras*

In this talk we will give a complete derived invariant for gentle algebras based on geometric surface models of their bounded derived categories. This invariant completes the invariant of Avella-Alaminos-Geiss. The latter is encoded solely in the information on the boundary components of the associated surface. We will see that in order to obtain a complete invariant, we also have to take into account the genus of the surface. This talk is based on joint work with Claire Amiot and Pierre-Guy Plamondon and on joint work with Sebastian Opper and Pierre-Guy Plamondon.

**Gordana Todorov** (Northeastern University)

*Continuous Quivers of Type A*

We consider quivers of continuous type A and prove basic results about pointwise finite dimensional representations, e.g. we prove analogue of the barcode theorem for the continuous quivers of type A with alternating orientations. Starting with this, we define a generalization of the continuous cluster categories. These categories have several new features: continuous clusters and continuous mutations, unlike the original continuous cluster categories. joint work with Kiyoshi Igusa, Job Rock.

**Changchang Xi** (Capital Normal University)

*Derived equivalences and algebraic K-groups of algebras*

We shall first present methods to construct both derived equivalences of algebras and recollements of derived module categories. We then apply these methods to studying algebraic K-theory of rings. Especially we get reduction formulas for higher algebraic K-groups of rings, including matrix subrings and the endomorphism rings of modules, from both derived equivalences and recollements of derived module categories.

**Pu Zhang** (Shanghai Jiao Tong University)

*Exceptional cycles for perfect complexes over gentle algebras*

Let $A$ be an indecomposable gentle algebra over an algebraically closed field $k$ with $A \neq k$. An component of the Auslander-Reiten quiver of $K^b(A-\mathrm{proj})$ is a characteristic component, if it contains a string complex at the mouth. Thus a characteristic component is of the form $\mathbb{Z}\mathbb{A}_{\ltimes}$ $(n \geq 2)$, $\mathbb{Z}\mathbb{A}_{\infty}$, $\mathbb{Z}\mathbb{A}_{\infty}/\langle \tau^{\ltimes} \rangle$ $(n \geq 1)$. The Hom spaces from string complexes at the mouth are explicitly determined. As a consequence, indecomposable objects in different characteristic components (in the sense of up to shift) are orthogonal. Also, there are no non-zero morphisms from string complexes in characteristic components to band complexes. We classify "almost all" the exceptional cycles (in the sense of Broomhead-Pauksztello-Ploog) in $K^b(A-\mathrm{proj})$, except those exceptional 1-cycles (spherical objects) which are band complexes. Namely, the mouth of each characteristic component of $K^b(A-\mathrm{proj})$ forms an exceptional cycle; if the quiver of $A$ is not of type $A_3$, this gives all the exceptional $n$-cycle in $K^b(A-\mathrm{proj})$ with $n \geq 2$, up to shift at each position and up to rotation; and a string complex is an exceptional 1-cycle if and only if it is at the mouth of a characteristic component with AG-invariant $(1, m)$. However, a band complex at the mouth is possibly an exceptional 1-cycle, and possibly not. The actions of the twist functors induced by exceptional cycles are explicitly given. In particular, all these twist functors preserve the Auslander-Reiten components.

This is a joint work with Peng Guo.