

Title and Abstract for Jean-Pierre Demailly Workshop

1. Principle Speaker : Jean-Pierre Demailly, Université Grenoble Alpes, Institut Fourier, France

Title: Ricci curvature and geometry of compact Kähler varieties

Abstract: The geometric structure of projective and compact Kähler manifolds is governed by the sign of their canonical bundle. We will recall here some fundamental results concerning (semi)positive line bundles and their link with the geometry of compact Kähler varieties. The main goal will be to describe fundamental structure theorems for compact Kähler manifolds with semipositive or nef anticanonical bundles.

Lecture 1: Positivity concepts in Kähler geometry

Abstract: We will first recall the basic definitions and characterizations of the concept of ample, nef, big and pseudoeffective line bundles and $(1,1)$ cohomology classes, in the context of projective, resp. Kähler varieties. The main duality results for positive cones will be described, and as a consequence, a necessary and sufficient criterion for rational connectedness will be established.

Lecture 2: A generalized holonomy principle and the De Rham and Cheeger-Gromoll splitting theorems

Abstract: The concept of holonomy of a euclidean or hermitian vector bundle will be explained, along with the De Rham splitting theorem and Berger's classification of holonomy groups. We will then introduce our recent result on the holonomy of certain holomorphic vector bundles whose curvature tensor has a trace that is a seminegative hermitian endomorphism.

Lecture 3: Structure theorems for compact Kähler manifolds with nef anticanonical bundles

Abstract: The results previously established will be combined to obtain a precise decomposition theorem for compact Kähler manifolds with semipositive Ricci curvature (following work of Campana, Peternell and the lecturer), after giving a presentation of the main examples. The case when K_X is nef will also be considered, as an introduction to the work of Junyan Cao and Andreas Höring.

2. Junyan Cao, Université Paris VI, Institut de Mathématiques de Jussieu, France

Title: Structure theorem for projective manifolds with nef anticanonical bundles

Abstract: The goal of our talks is to present joint work with Andreas Höring on the structure of projective manifolds with nef anticanonical bundles. We prove that a simply connected such manifold is a product of a rationally connected manifold and a manifold with trivial canonical bundle. As an application we describe the MRC fibration of any projective manifold with nef anticanonical bundle.

3. Jian Xiao, Northwestern University, USA

Title: Local positivity for curves

Abstract: One of the most important invariants measuring the local positivity of a nef line bundle is the local Seshadri function introduced by Demailly. We first give a brief introduction to this invariant. Then using the duality of positive cones, we show that applying the polar transform from convex analysis to local positivity invariants for divisors gives interesting and new local positivity invariants for curves. These new invariants have nice properties similar to those for divisors. In particular, this enables us to obtain a Seshadri type ampleness criterion for movable curves, and give a characterization of the divisorial components of the non-Kähler locus of a big class. (Joint work with N. McCleerey.)

4. Xiaokui Yang, Morningside Center of Mathematics, Chinese Academy of Sciences, China

Title: RC positivity, rational connectedness and Yau's conjecture

Abstract: In this presentation, we will describe the relationship between various positivity notions in differential geometry and algebraic geometry. We shall also introduce a new concept called "RC-positivity" in differential geometry and use it to characterize uniruled and rationally connected projective manifolds. In particular, we confirm a conjecture of Yau that a compact Kähler manifold with positive holomorphic sectional curvature is projective algebraic and rationally connected.

5. Xiangyu Zhou, Institute of Mathematics, Chinese Academy of Sciences, China

Title: Some applications of the optimal L^2 extension and strong openness of multiplier ideal sheaves

Abstract: In this talk, we will first recall our solution of the optimal L^2 extension problem and of Demailly's strong openness conjecture on multiplier ideal sheaves. We will then present some applications and consequences of these results.