Titles and Abstracts

A virtual element method for elliptic equations in nondivergence form

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The numerical discretisation of problems in nondivergence form is notoriously challenging due to the lack of notion of weak solutions based on variational principles. We consider the model problem of nondivergence form elliptic equations arising from the linearization of fully nonlinear equations, such as the Hamilton-Jacobi-Bellman equation. The model problem is well-posed under some condition on the coefficients, called the Cordes condition. The proof relies on the Miranda-Talenti inequality, which bounds the L^2 norm of the Hessian of H^2 functions with zero trace on convex domains by their laplacian. We exploit the availability of H^2 conforming Virtual Element spaces to design a Virtual Element Method (VEM) based on an equivalent variational problem. We show that the stability of this method follows directly from the availability of the Miranda-Talenti inequality in H^2 conforming spaces, and deduce that the method converges optimally in the H^2 norm. Surprisingly for a VEM, and thanks to the specific variational formulation used, error estimates do not involve the regularity of the data. This extends to the analysis of the effect of numerical quadrature. We illustrate those results by some numerical experiments.

A nonlinear least-squares convexity enforcing finite element method for the Monge-Ampere equation

Susanne C. Brenner
Louisiana State University, America

We present a nonlinear least-squares finite element method for computing the smooth convex solutions of the Dirichlet boundary value problem of the Monge-Ampere equation on smooth strictly convex planar domains.   It is based on an isoparametric finite element space with exotic degrees of freedom that can enforce the convexity of the approximate solutions.

Orthogonality preserving schemes for electronic structure calculations

Xiaoying Dai
Academy of Mathematics and Systems Science，CAS

To obtain convergent numerical approximations without orthogonalization operations is of great importance in electronic structure calculations. In this talk, we will introduce an extended gradient flow based Kohn-Sham DFT model, for which we prove the flow is orthogonality preserving and the solution evolves to the ground state. With the help of the gradient flow based Kohn-Sham DFT model, we propose and analyze a class of iteration schemes for the discretized Kohn-Sham model, which preserves the orthogonality of the Kohn-Sham orbitals automatically. With our schemes, the iterative approximations are guaranteed to converge to the Kohn-Sham orbitals without any orthogonalization operations when the initial orbitals are orthogonal. We prove the convergence and get the local convergence rate of the numerical approximations under some reasonable assumptions. Besides, based on the adaptive finite element discretization, we apply these schemes to the electronic structure calculation of some typical systems, and the numerical experiments validate our theoretical results. This is a joint work with Qiao Wang, Liwei Zhang and Aihui Zhou.

Sobolev stability of the L^2-projection

Lars Diening
Mathematisches Institut der University München

We prove the $W^{1,2}$-stability of the L^2-projection on Lagrange elements for adaptive meshes and arbitrary polynomial degree. This property is especially important for the numerical analysis of parabolic problems. We will explain that the stability of the projection is connected to the grading constants of the underlying adaptive refinement routine. For arbitrary dimensions we show that the bisection algorithm of Maubach and Traxler produces meshes with a grading constant 2. This implies $W^{1,2}$-stability of the L^2-projection up to dimension six. We show that the projection is also applicable for non-linear parabolic problems.

A posteriori error estimates for discontinuous Galerkin methods for the Allen-Cahn problem on polytopic meshes.

Zhaonan Dong
Inria Paris

We are concerned with the proof of a posteriori error (upper) bounds for fully-discrete Galerkin approximations of the Allen-Cahn equation in two and three spatial dimensions. We begin by discussing the case of the backward Euler method combined with conforming finite elements on standard meshes in space before continuing with the case of space-time discontinuous Galerkin methods on very general, polytopic, prismatic meshes. For both methods, we prove conditional type a posteriori error estimates in the L4(L4)-norm that depend polynomially upon the inverse of the interface length ε. The derivation relies on the availability of a spectral estimate for the linearized Allen-Cahn operator about the approximating solution in conjunction with a continuation argument and a variant of the elliptic reconstruction. The new analysis also improves variants of known a posteriori error bounds in L2(H1), L∞(L2)-norms in certain regimes.

Stochastic collocation for dynamic micromagnetism

Michael Feischl
Vienna technical university

We consider the stochastic Landau-Lifschitz-Gilbert equation, an SPDE model for dynamic micromagnetism. We first convert the problem to a (highly nonlinear) PDE with parametric coefficients using the Doss-Sussmann transform and the Levy-Ciesielsky parametrization of the Brownian motion. We prove analytic regularity of the parameter-to-solution map and estimate its derivatives. These estimates are used to prove convergence rates for piecewise polynomial sparse grid methods. Moreover, we propose novel time-stepping methods to solve the underlying deterministic equations Sampling widths of function classes

Unified finite element analysis for semilinear fourth-order problems with quadratic nonlinearity

Benedikt Grässle
Humboldt-Universität zu Berlin, Germany

The a priori and a posteriori error analysis in [1, 3] establishes a unified analysis for different finite element approximations to regular roots of nonlinear partial differential equations with a quadratic nonlinearity. A smoother in the source and nonlinearity enables quasi-best approximations in [3] under a set of hypotheses that guarantees the existence and local uniqueness of a discrete solutions by the Newton-Kantorovich theorem. Related assumptions on some computed approximation close to a regular root allow the reliable and efficient a posteriori error analysis [1] for a general class of rough sources introduced in [2]. Applications include nonconforming discretisations for the von K´arm´an plate and the stream-vorticity formulation of the stationary Navier-Stokes equations in 2D by the Morley, two versions of discontinuous Galerkin, C0 interior penalty, and WOPSIP methods. The talk presents joint work within the working groups of Prof. C. Carstensen and Prof. N. Nataraj.

[1] C. Carstensen, B. Gr¨aßle, and N. Nataraj. A posteriori error control for fourth-order semilinear problems with quadratic nonlinearity. SIAM J. Numer. Anal., arXiv:2309.08427, 2023.
[2] C. Carstensen, B. Gr¨aßle, and N. Nataraj. Unifying a posteriori error analysis of five piecewise quadratic discretisations for the biharmonic equation. J. Numer. Math., arXiv:2310.05648, 2023.
[3] C. Carstensen, N. Nataraj, G. C. Remesan, and D. Shylaja. Unified a priori analysis of four second-order FEM for fourth-order quadratic semilinear problems. Numer. Math., 154(3-4):323–368, Aug. 2023

A Construction of C^r Conforming Finite Element Spaces in Any Dimension

Jun Hu
Peking University

This talk proposes a construction of C^r conforming finite element spaces with arbitrary r in any dimension. It is shown that if $k \ge 2^{d}r+1$ the space P_k of polynomials of degree $\le k$ can be taken as the shape function space of C^r finite element spaces in d dimensions. This is the first work on constructing such C^r conforming finite elements in any dimension in a unified way.

Newton's method and its hybrid to machine learning for Navier-Stokes Darcy models Discretized by mixed element methods

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Navier-Stokes Darcy (NSD) models are frequently encountered in various industrial and engineering applications. In this talk, we are concerned with proposing and analyzing Newton's method and its hybrid to machine learning for solving a nonlinear discrete problem arising from mixed element discretization of the NSD model. At the beginning, Newton's method is designed for such a discrete problem. Under some standard conditions, the method is proved to be convergent quadratically, with the convergence rate independent of the finite element mesh size. To further improve the performance of the previous method, we use the interpolant of the PINN solution of the NSD model as the initial guess, so as to produce a hybrid method associated with machine learning for the previous problem. Numerical results show that the hybrid method performs better over Newton's method by choosing a standard initial guess. This is a joint work with Hui Peng and Haohao Wu from Shanghai Jiao Tong University.

Error analysis for a local discontinuous Galerkin approximation for systems of p-Navier--Stokes type

Alex Kaltenbach
Institute of Mathematics, Technical University of Berlin

In this talk, we propose a Local Discontinuous Galerkin (LDG) approximation for systems of p-Navier—Stokes type involving a new numerical flux in the stabilization term and a new discretization of the convective term. A priori error estimates are derived for the velocity, which are optimal for all p>2 and $\delta \geq 0$. A new criterion is presented that yields a priori error estimates for the pressure, which are optimal for all p>2 and $\delta \geq 0$.

Convergence of evolving finite element approximations to surface/interface/boundary evolution

Buyang Li

The Hong Kong Polytechnic University

We briefly review the development of evolving finite element methods for PDEs on evolving surfaces, and the evolving finite element approximations to solution-driven surface evolution. Then we report some recent applications of this approach to evolving finite element approximations to shape optimization and two-phase fluid flows.

Supercloseness and Asymptotic Analysis for the nonconforming Crouzeix-Raviart element

Limin Ma
Wuhan University

In this talk, we will discuss the supercloseness and the superconvergence of the Crouzeix-Raviart and enriched Crouzeix-Raviart elements for the Poisson equation and the Stokes equation on uniform triangulations. Two pseudostress interpolations are designed and proved to admit a full one-order supercloseness. A superconvergence result is proved for these nonconforming elements by applying the supercloseness and an appropriate postprocessing technique. Moreover, for the eigenvalues of the Laplace operator and the Stokes operator by these two nonconforming elements, an application of the supercloseness leads to an intrinsic and concise asymptotic analysis of numerical eigenvalues, which proves an optimal superconvergence of eigenvalues by the extrapolation algorithm. Finally, numerical experiments are tested to verify the theoretical results.

A new mixed finite element method for the Cahn-Hilliard equation on triangular and tetrahedral grids

Rui Ma
Beijing Institute of Technology

This talk will present a new fully discrete semi-implicit mixed finite element method based on the mixed finite elements from [Hu, Ma and Zhang, Sci. China Math. 2021] to solve the Cahn-Hilliard equation in two and three dimensions. The new mixed method suits for non-convex domains. The well-posedness and the error estimates are provided. This talk will give some numerical results to show the efficiency and accuracy of the proposed method.

Adaptive multi-level algorithm for a class of nonlinear problems

Eun-Jae Park
Yonsei University

In this talk, we first develop and analyze two-grid/multi-level algorithms via mesh refinement in the abstract framework of Brezzi, Rappaz, and Raviart for approximation of branches of nonsingular solutions. Optimal fine grid accuracy of two-grid/multi-level algorithms can be achieved via the proper scaling of relevant meshes. An important aspect of the proposed algorithm is the use of mesh refinement in conjunction with Newton-type methods for system solution in contrast to Newton's method on a fixed mesh. Then, we propose an adaptive mesh-refining based on the multi-level algorithm and derive a unified a posteriori error estimate for a class of nonlinear problems. We have shown that the multi-level algorithm on adaptive meshes retains quadratic convergence of Newton's method across different mesh levels, which is numerically validated. Our framework facilitates to use the general theory established for a linear problem associated with given nonlinear equations. In particular, existing a posteriori error estimates for the linear problem can be utilize ed to find reliable error estimators for the given nonlinear problem.As applications of our theory, we consider the pseudostress-velocity formulation of Navier-Stokes equations and the standard Galerkin formulation of semilinear elliptic equations. Reliable and efficient a posteriori error estimators for both approximations are derived. Finally, several numerical examples are presented to test the performance of the algorithm and validity of the theory developed.

Homogenization of Hamilton--Jacobi equations: optimal rates and numerical effective Hamiltonians

Timo Sprekeler
National University of Singapore

First, we discuss the optimal rate of convergence in periodic homogenization of viscous Hamilton--Jacobi equations, and the numerical approximation of the effective Hamiltonian. The numerical scheme is based on a finite element approximation of approximate corrector problems for Hamilton--Jacobi--Bellman (HJB) Hamiltonians. Thereafter, we study the approximation of the effective Hamiltonian corresponding to second-order HJB and Isaacs Hamiltonians in a framework surrounding a Cordes-type condition. This is based on joint works with E. Kawecki (Mahindra Racing), J. Qian (Michigan), H.V. Tran (Wisconsin), and Y. Yu (Irvine).

Kacanov iterations for the p-Laplacian and modifications for minimal residual methods in $W^{-1,p}$

Johannes Storn
Universität Bielefeld

The main part of this talk introduces an iterative scheme for the computation of the discrete minimizer to the p-Laplace problem. The iterative scheme is easy to implement since each iterate results only from the solve of a weighted, linear Poisson problem. It neither requires an additional line search nor involves unknown constants for the step length. The scheme converges globally and its rate of convergence is independent of the underlying mesh. In the second part of the talk we adjust this ansatz to compute the minimizer of a minimal residual method in W^{-1,p}. Such schemes remedy instabilities of finite element methods for problems like convection-dominated diffusion.

A finite element method for a two-dimensional Pucci equation

Zhiyu Tan
School of Mathematical Sciences, Xiamen University

A nonlinear least-squares finite element method for strong solutions of the Dirichlet boundary value problem of a two-dimensional Pucci equation on convex polygonal domains is investigated in this paper. We obtain a priori and a posteriori error estimates and present corroborating numerical results, where the discrete nonsmooth and nonlinear optimization problems are solved by an active set method and an alternating direction method with multipliers.

High-order accurate entropy stable nodal discontinuous Galerkin schemes for the special relativistic magnetohydrodynamics

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High-order accurate entropy stable nodal discontinuous Galerkin (DG) schemes are studied for the ideal special relativistic magnetohydrodynamics (RMHD). They are built on the modified RMHD equations with a particular source term, which is analogous to the Powell's eight-wave formulation and can be symmetrized so that an “entropy pair” is obtained. We design an affordable “fully consistent” two-point entropy conservative flux, which is not only consistent with the physical flux, but also maintains the zero parallel magnetic component, and then construct high-order accurate semi-discrete entropy stable DG schemes based on the quadrature rules and the entropy conservative and stable fluxes. The Lax-Friedrichs flux is proven to be entropy stable when its viscosity coefficient is chosen as the speed of light, and the other two entropy stable fluxes are also derived by adding the dissipation terms to our proposed entropy conservative flux and compared to the Lax-Friedrichs flux by using numerical experiments in terms of the CPU time and accuracy. All three entropy stable numerical fluxes maintain the zero parallel magnetic component in one dimension. The resulting entropy stable DG schemes satisfy the semi-discrete “entropy inequality” for the given “entropy pair” and are integrated in time by using the high-order explicit strong stability preserving Runge-Kutta schemes to get further the fully-discrete nodal DG schemes. Extensive numerical tests are conducted to validate the accuracy and the ability to capture discontinuities of our schemes with the help of the TVB limiter. Moreover, our entropy conservative flux is compared to an existing flux through some numerical tests. The results show that the zero parallel magnetic component in the numerical flux can help to decrease the error in the parallel magnetic component in one-dimensional tests, but two entropy conservative fluxes give similar results since the error in the magnetic field divergence seems dominated in the two-dimensional tests.

Regularization scheme for the Monge-Amper\`e equation in 2d.

Tran Ngoc Tien
University of Jena

This talk is devoted to a regularization of the Monge–Amp`ere equation in planar convex domains through uniformly elliptic Hamilton–Jacobi–Bellman equations. The regularized problem possesses a unique strong solution $u\varepsilon$ and is accessible to the discretization with finite elements. We establish uniform convergence of $u\varepsilon$ to the convex Alexandrov solution u of the Monge–Amp`ere equation as the regularization parameter ε approaches 0 with convergence rates in ε under additional assumptions on the right-hand side. The Alexandrov maximum principle allows for an a posteriori error estimator for conforming methods, which motivates adaptive mesh-refining algorithms. Numerical experiments are presented for a comparison of different mesh-refining strategies.

Finite Element Method for a Nonlinear Perfectly Matched Layer Helmholtz Equation with High Wave Number

Haijun Wu
Nanjing University

A nonlinear Helmholtz equation (NLH) with high wave number and Sommerfeld radiation condition is approximated by the perfectly matched layer (PML) technique and then dis- cretized by the linear finite element method (FEM). Wave-number-explicit stability and regularity estimates and the exponential convergence are proved for the nonlinear truncated PML problem. Preasymptotic error estimates are obtained for the FEM where the logarithmic factors in h required by the previous results for the NLH with impedance boundary condition are removed in the case of two dimensions. Moreover local quadratic convergences of the Newton's methods are derived for both the NLH with PML and its FEM. Numerical examples are presented to verify the accuracy of the FEM which demonstrate that the pollution errors may be greatly reduced by applying the interior penalty technique with proper penalty parameters to the FEM. The nonlinear phenomenon of optical bistability can be successfully simulated.

Monotone Discretization of Integral Fractional Laplacian on Bounded Lipschitz Domains: Applications to the Fractional Obstacle Problem

Shuonan Wu
Peking University

We propose a monotone discretization method for the integral fractional Laplacian on bounded

Lipschitz domains with homogeneous Dirichlet boundary conditions, specifically designed for solving fractional obstacle problems. Operating on unstructured grids in arbitrary dimensions, the method offers flexibility in approximating singular integrals over a domain that depends not only on the local grid size but also on the distance to the boundary, where the H\"older coefficient of the solution deteriorates. Using a discrete barrier function reflecting the distance to the boundary, we demonstrate optimal pointwise convergence rates in terms of the H\"older regularity of the data on quasi-uniform and graded grids. Applying this monotone discretization to the (nonlinear) fractional obstacle problems, we establish the uniform boundedness, existence, and uniqueness of numerical solutions. Monotonicity naturally implies the convergence of the policy iteration. Subsequently, based on the nature of this problem, an improved policy iteration tailored to solution regularity is devised, exhibiting superior performance through adaptive discretization across diverse regions. Several numerical examples are provided to illustrate the sharpness of the theoretical results and the efficacy of the proposed method.

Augmented subspace method and its applications

Hehu Xie
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In this talk, we introduce a type of augmented subspace method and its application in solving eigenvalue problems and semilinear equations. The augmented subspace is built with a coarse finite element space and an finite element function in the fine mesh. Based on the augmented subspace, we can transforming solving the eigenvalue problem in the fine finite element space to the linear solving in the fine finite element space and an small scale eigenvalue problem on the low dimensional augmented subspace. This property improves overfull efficiency for solving eigenvalue problems. Here, the corresponding theoretical analysis and numerical applications will be introduced.

Time-domain mathematical modeling, finite element simulation, and design in complex anisotropic electromagnetic media

Wei Yang
Xiangtan University

In this talk, we present a time-domain mathematical model, finite element methods, and related numerical theories for the electromagnetic propagation problem in three-dimensional anisotropic media. The model is capable of simultaneously characterizing the linear and nonlinear properties of materials. In terms of linear properties, we employ a designed finite element method to create five types of hyperbolic metamaterials within the 50-400nm wavelength range, and numerically verify their optical performance across a wide frequency spectrum. Additionally, by combining linear hyperbolic metamaterials with nonlinear materials, we simulate and design a nonlinear hyperbolic metamaterial, effectively enhancing the third harmonic generation of electromagnetic materials using hyperbolic materials.

A priori and a posteriori error analysis of mixed DG methods for the quasi-Newtonian Stokes flow

Lina Zhao
The Hong Kong Polytechnic University

In this talk, I will present a new mixed-type DG method for the three-field quasi-Newtonian Stokes flow. The scheme is based on the introduction of the stress and strain tensor as further unknowns, as well as the elimination of the pressure variable by means of the incompressibility constraint. As such, the resulting system involves three unknowns: the stress, the strain tensor, and the velocity. All these three unknowns are approximated using discontinuous piecewise polynomials, which offer flexibility for enforcing the symmetry of the stress and the strain tensor. The unique solvability and a comprehensive convergence error analysis for all the variables are performed. All the variables are proven to converge optimally. Adaptive mesh refinement guided by a posteriori error estimator is computationally efficient, especially for problems involving singularity. In line with this mechanism, we derive a residual-type a posteriori error estimator, which constitutes the second main contribution of the paper. In particular, we employ the elliptic reconstruction in conjunction with the Helmholtz decomposition to derive the a posteriori error estimator, which avoids using the averaging operator. Several numerical experiments are carried out to verify the theoretical findings.