Titles and Abstracts

Uniform error bounds on numerical methods for long-time dynamics of dispersive PDEs

Weizhu Bao(包维柱)
National University of Singapore

In this talk, I report our recent work of error estimates on different numerical methods for the long-time dynamics of dispersive PDEs with small potential or weak nonlinearity, such as the Schroedinger equation with small potential, the nonlinear Schroedinger equation with weak nonlinearity, the nonlinear Klein-Gordon equation with weak nonlinearity, the Dirac equation with small electromagnetic potential, and the nonlinear Dirac equation with weak nonlinearity, etc.

By introducing a new technique of regularity compensation oscillatory (RCO), we can establish improved uniform error bounds on time-splitting methods for dispersive PDEs with small potentials and/or weak nonlinearity.

This talk is based on joint works with Yongyong Cai and Yue Feng.

Convergence of evolving finite element approximations to surface/interface/boundary evolution

Buyang Li(李步扬)
The Hong Kong Polytechnic University

We briefly review the development of evolving finite element methods for PDEs on evolving surfaces, and the evolving finite element approximations to solution-driven surface evolution. Then we report some recent applications of this approach to evolving.

Random ordinate method for mitigating the ray effect in radiative transport equation simulations

Min Tang(唐敏)
Shanghai Jiao Tong University

The Discrete Ordinates Method (DOM) is the most widely used velocity discretization method for simulating the radiative transport equation. The ray effect stands as a long-standing drawback of DOM. In benchmark tests displaying the ray effect, we observe low regularity in velocity within the solution. To address this issue, we propose a random ordinate method (ROM) to mitigate the ray effect. Compared with other strategies proposed in the literature for mitigating the ray effect, ROM offers several advantages: 1) the computational cost is comparable to DOM; 2) it is simple and requires minimal changes to existing code based on DOM; 3) it is easily parallelizable and independent of the problem setup. Analytical results are presented for the convergence orders of the error and bias, and numerical tests demonstrate its effectiveness in mitigating the ray effect.

Parallel-in-time solver for a high-order BDF discretization

Zhi Zhou(周知)
The Hong Kong Polytechnic University

In this talk, we propose and analyze a parallel-in-time algorithm for approximately solving parabolic equations. We apply the k-step backward differentiation formula, and then develop an iterative solver by using the waveform relaxation technique. Each resulting iterate represents a periodic-like system, which could be further solved in parallel by using the diagonalization technique. The convergence of the waveform relaxation iteration is theoretically examined by using the generating function method. The argument could be further applied to the time-fractional subdiffusion equation, whose discretization shares common properties of the standard BDF methods, because of the nonlocality of the fractional differential operator. Some illustrative numerical results will be presented to complement the theoretical analysis.

Optimal zero-padding of kernel truncation method

Yong Zhang(张勇)
Tianjin University

The kernel truncation method (KTM) is a commonly-used algorithm to compute the convolution-type nonlocal potential, where the convolution kernel might be singular at the origin and/or far-field and the density is smooth and fast-decaying. In KTM, in order to capture the Fourier integrand's oscillations that is brought by the kernel truncation, one needs to carry out a zero-padding of the density, which means a larger physical computation domain and a finer mesh in the Fourier space by duality. The empirical fourfold zero-padding [Vico et al. J. Comput. Phys. (2016)] puts a heavy burden on memory requirement especially for higher dimension problems. In this paper, we derive the optimal zero-padding factor, that is, \sqrt{d}+1, for the first time together with a rigorous proof. The memory cost is greatly reduced to a small fraction, i.e., (\frac{\sqrt{d}+1}{4})^d, of what is needed in the original fourfold algorithm.

For example, in the precomputation step, a double-precision computation on a 256^3 grid requires a minimum 3.4 Gb memory with the optimal threefold zero-padding, while the fourfold algorithm requires around 8 Gb where the reduction factor is around 60%. Then, we present the error estimates of the potential and density in d space dimension. Next, we re-investigate the optimal zero-padding factor for the anisotropic density. Finally, extensive numerical results are provided to confirm the accuracy, efficiency, optimal zero-padding factor for the anisotropic density, together with some applications to different types of nonlocal potential, including the 1D/2D/3D Poisson, 2D Coulomb, quasi-2D/3D Dipole-Dipole Interaction and 3D quadrupolar potential.

Accelerated energy minimizing flows for large bending deformations of nonlinear plates

Shuo Yang(杨朔)
Beijing Institute of Mathematical Sciences and Applications

In recent years, there has been some numerical explorations of large bending deformations in nonlinear plates, including single layer, prestrained, and bilayer plates. The mathematical problems consist of minimizations of bending energy functionals while subject to nonlinear and non-convex

metric constraints. In previous works, the computation of such non-convex constrained energy minimization problems has relied on gradient flows. While these approaches are known for their stability, they often exhibit slow convergence, especially when dealing with highly non-convex problems. In this presentation, we introduce new iterative schemes that surpass existing gradient flow algorithms in terms of speed and efficiency. In our accelerated flows, we linearize the metric constraint through incremental updates to a tangent plane, building upon the previous iteration. Rigorous analysis confirms the stability of these accelerated flows, and also proves the proper control over constraint violations. Furthermore, we discuss other related properties, including the convergence of flow.

The ultraspherical spectral method

Kuan Xu(徐宽)
University of Science and Technology of China

超球谱方法（Ultraspherical spectral method）是一种新诞生的求解各类微分方程、积分方程的谱方法，具有速度快、精度高、稳定性强、易自适应的特点。本报告将介绍这一方法和它最近的发展。

Advancements in Robust Bound-Preserving MUSCL-Hancock Schemes

Guoxian Chen(陈国贤)
Wuhan University

This lecture explores novel stability conditions for the MUSCL-Hancock upwind scheme, a widely utilized approach for solving hyperbolic conservation laws. Contrary to existing stability criteria, we introduce bound-preserving conditions, allowing for a CFL number of $(\sqrt{3}-1)/2$ enabling faster simulations. Additionally, we propose a slope corrector for preliminary reconstruction, ensuring bound-preserving approximations globally and non-oscillatory behavior locally. The corrected scheme, when applied with a generalized minmod limiter, demonstrates enhanced stability and robustness in various scenarios, including advection problems with spatial variability and general nonlinear systems. Numerical examples validate the efficacy of these innovations, showcasing improved speed and resolution capabilities.

Traceability of Water Pollution: An Inversion Scheme via Dynamic CGO Solutions

Lingyun Qiu(邱凌云)
Tsinghua University

We aim to find the time-dependent source term in the diffusion equation from the boundary measurement, which allows for the possibility of tracing back the source of pollutants in the environment. Based on the idea of dynamic complex geometrical optics (CGO) solutions, we analyze a variational formulation of the inverse source problem and prove the uniqueness and stability result. A two-step reconstruction algorithm is proposed, which first recovers the locations of the point sources, and then the Fourier components of the emission concentration functions are reconstructed. Numerical experiments on simulated data are conducted. The results demonstrate that our proposed two-step reconstruction algorithm can reliably reconstruct multiple point sources and accurately reconstruct the emission concentration functions. In addition, we decompose the

algorithm into two parts: online and offline computation, with most of the work done offline. This paves the way towards real-time traceability of pollution. The proposed method can be used in many fields, particularly those related to water pollution, to identify the source of a contaminant in the environment and can be a valuable tool in protecting the environment.

Improved Uniform Error Bounds on Time-splitting Methods for Long-time Dynamics of Dispersive PDEs

Yue Feng(冯悦)
Xi'an Jiaotong University

In this talk, I begin with the nonlinear Klein-Gordon equation (NKGE) with weak nonlinearity, which is characterized by ε^2 with ε ∈ (0,1] a dimensionless parameter. Different numerical methods are applied to discretize the NKGE including finite difference methods, exponential wave integrators and time-splitting methods. Especially, we discretize the NKGE by the second-order time-splitting method in time and combine with the Fourier spectral method in space. By introducing a new technique--Regularity Compensation Oscillation (RCO) which controls the high frequency modes by the regularity of the exact solution and analyzes the low frequency modes by phase cancellation and energy method, we carry out the improved uniform error bounds for the time-splitting methods. The results have been extended to other dispersive PDEs including the (nonlinear) Schrodinger equation and Dirac equation.

High-order in time, BGN-based parametric finite element methods for solving geometric flows

Wei Jiang(蒋维)
Wuhan University

Geometric flows have recently attracted lots of attention from scientific computing communities. One of the most popular schemes for solving geometric flows is the so-called BGN scheme, which was proposed by Barrett, Garcke, and Nurnberg (J. Comput. Phys., 222 (2007), pp. 441--467). However, the BGN scheme only can attain first-order accuracy in time, and how to design a temporal high-order numerical scheme is challenging. Recently, based on a novel approach, we have successfully proposed temporal high-order, BGN-based parametric finite element method for solving geometric flows of curves/surfaces. Furthermore, we point out that the shape metrics (i.e., manifold distance), instead of the function norms, should be used to measure numerical errors of the proposed schemes. Finally, ample numerical experiments demonstrate that the proposed BGN-based schemes are high-order in time in terms of the shape metric, and much more efficient than the classical BGN schemes. This is a joint work with Chunmei Su and Ganghui Zhang.

On splitting methods for the Dirac equation in the nonrelativistic limit regime

Yongyong Cai(蔡勇勇)
Beijing Normal University

We establish error bounds of the Lie-Trotter splitting and Strang splitting for the Dirac equation in the nonrelativistic limit regime in the absence of external magnetic potentials. In this regime, the solution admits high frequency waves in time. Surprisingly, we find out that the splitting methods

exhibit super-resolutions, i.e. the methods can capture the solutions accurately even if the time step size is much larger than the sampled wavelength. We also discuss the case with external magnetic potentials, and splitting schemes also show superior performance among the commonly used numerical methods.

Asymptotic preserving SL-DG methods for multiscale kinetic transport equations

Tao Xiong(熊涛)
Xiamen University

In this work, we have formulated a class of asymptotic-preserving (AP) semi-Lagrangian (SL) discontinuous Galerkin (DG) methods, designed for kinetic transport equations under a diffusive scaling. These methods partition the problem into two components: macroscopic (density) and microscopic (distribution function). We solve the density equation by employing a semi-Lagrangian discontinuous Galerkin (SL DG) method for the transport term and an implicit Local Discontinuous Galerkin (LDG) method for the diffusive term. We employ another SL DG method, incorporating a discrete velocity approximation for the distribution function. Once the density is determined, the distribution function becomes completely decoupled in terms of velocity, allowing for efficient parallel computing without concerns about data racing. Numerical experiments, covering one-dimensional to three-dimensional problems, confirm the accuracy, AP property, stability, and parallel efficiency of the proposed methods.

This is a joint work with Yi Cai, Guoliang Zhang and Hongqiang Zhu.

A moment-based Hermite WENO scheme with unified stencils for hyperbolic conservation laws

Zhuang Zhao(赵状)
Xiamen University

In this talk, we introduce a fifth-order moment-based Hermite weighted essentially non-oscillatory scheme with unified stencils (termed as HWENO-U) for hyperbolic conservation laws. The main idea of the HWENO-U scheme is to modify the first-order moment by a HWENO limiter only in the time discretizations using the same information of spatial reconstructions, in which the limiter not only overcomes spurious oscillations well, but also ensures the stability of the fully-discrete scheme. For the HWENO reconstructions, a new scale-invariant nonlinear weight is designed by incorporating only the integral average values of the solution, which keeps all properties of the original one while is more robust for simulating challenging problems with sharp scale variations. Compared with previous HWENO schemes, the advantages of the HWENO-U scheme are: (1) a simper implemented process involving only a single HWENO reconstruction applied throughout the entire procedures without any modifications for the governing equations; (2) increased efficiency by utilizing the same candidate stencils, reconstructed polynomials, and linear and nonlinear weights in both the HWENO limiter and spatial reconstructions; (3) reduced problem-specific dependencies and improved rationality, as the nonlinear weights are identical for the function u and its non-zero multiple ζu. Besides, the proposed scheme retains the advantages of previous HWENO schemes, including compact reconstructed stencils and the utilization of artificial linear weights. Extensive benchmarks are carried out to validate the accuracy, efficiency, resolution, and robustness of the proposed scheme.

A theoretical framework for moving finite element method by the Onsager principle

Xianmin Xu(许现民)
Chinese Academy of Sciences, LSEC

In this talk, we will present an application the Onsager variational principle on numerical analysis for moving finite element method (MFEM). We give a novel derivation for the MFEM for gradient flow equations. We show that the discretized problem has the same energy dissipation structure as the continuous one. This enables us to do numerical analysis for the stationary solution of a nonlinear reaction diffusion equation using the approximation theory of free-knot piecewise polynomials. We show that under certain conditions the solution obtained by the moving finite element method converges to a local minimizer of the total energy when time goes to infinity. The global minimizer, once it is detected by the discrete scheme, approximates the continuous stationary solution in optimal order. For the dynamical problem, we introduce a regularized metric to the finite element space with free knots. This leads to a smooth Riemann manifold and a metric space when considering geodesic distance. We show that the moving finite element scheme for a nonlinear reaction-diffusion equation can be viewed as a curve of maximal slope in the discrete metric space. This enables us to propose a JKO scheme and an explicit stabilized numerical scheme for the moving finite element method. We present some proof for the convergence of the method by using the gradient flow theory in metric spaces. Numerical examples are given to show that the discrete schemes work well in both one dimensional and two dimensional cases.

Active Learning of Transition State of Free Energy

Xiang Zhou(周翔)
City University of Hong Kong

In the study of rare event, transition state is one of central concepts and in mathematics, which can regarded as a saddle point (SP) search problem. This talk will review the works in my group about the methods of finding these saddle points, including the multiscale gentlest ascent dynamics and the iterative minimization algorithms. The main focus is then on the new development of the methods for computationally intensive function such as the free energy surface by adopting the recent development of machine learning techniques. This method combines a statistical method, Gaussian process regression (GPR) and the Gentlest Ascent Dynamics. We sequentially detect the SP by GAD applied to the surrogate model and update the surrogate GPR by an active learning strategy based on the design of experiment so that the uncertainty of the trained surrogate model is minimized and the efficiency gets a significant improvement. This is joint work with Dr Hongqiao Wang and Dr Shuting Gu.

Efficient threshold dynamics methods for topology optimization for fluids and heat transfer problems

Huangxin Chen(陈黄鑫)
Xiamen University

In this talk, we will introduce an efficient threshold dynamics method for topology optimization for fluids modeled with the Stokes equation. A one-domain approach is applied to solve the problem in the whole domain and the minimization problem can be solved with an iterative

scheme. The total energy decaying property of the iterative algorithm can be obtained. The extensions of the iterative thresholding method will also be introduced for topology optimization for the Navier-Stokes flow and the heat transfer problems.

Some economically high-order numerical schemes

Shuo Zhang(张硕)
Chinese Academy of Sciences, LSEC

The accuracy (order of the convergence rate) of a numerical schemes depends on both the degree of the polynomial space for approximation and the smoothness of the functions to be approximated. If a scheme uses the lowest-degree polynomials ever possible with respect to the accuracy, we call this scheme an economically high-order scheme. This talk introduces some economically high-order schemes, associated with the fundamental model problems such as the Poisson equation, the biharmonic equation and the Stokes problem. The schemes reported here fall into a nonstandard category of finite element methods, called the non-Ciarlet type finite element method. Motivated by the construction of low-degree schemes, some unexpected numerical phenomena are also revealed, essentially useful in solving both boundary value problems and eigenvalue problems.

Surface reconstruction based modified Gauss formula

Zuoqiang Shi(史作强)
Tsinghua University

Surface reconstruction aims at reconstructing continuous surface from discrete point cloud. This is a fundamental problem in computer vision, 3D modeling and many other applications. In this talk, we introduce several surface reconstruction methods based famous Gauss formula in potential theory. Gauss formula provide an explicit integral formula for indicator function. Then surface reconstruction can be transformed to be an integral over the surface which can be computed efficiently by proper quadrature rule and fast multipole method (FMM). For point cloud without oriented normals, Gauss formula also gives effective reconstruction in an implicit manner. Extensive experiments show that our methods are very effective and efficient, even outperform learning based methods.

IB-UQ: Information bottleneck based uncertainty quantification for neural function regression and neural operator learning

Wenwen Zhou(周雯雯)
Shanghai Normal University

In this talk, we will present a novel framework for uncertainty quantification via information bottleneck (IB-UQ) in scientific machine learning tasks, including deep neural network (DNN) regression and neural operator learning(DeepONet). IB-UQ can provide both mean and variance in the label prediction by explicitly modeling the representation variables. Compared to most DNN regression methods and the deterministic DeepONet, the proposed model can be trained on noisy data and provide accurate predictions with reliable uncertainty estimates on unseen noisy data. The capability of the proposed IB-UQ framework is demonstrated with some numerical

examples.

Entropy structure informed learning for solving inverse problems of differential equations

Liu Hong(洪柳)
Sun Yat-Sen Universtiy

Entropy, since its first discovery by Ludwig Boltzmann in 1877, has been widely applied in diverse disciplines, including thermodynamics, continuum mechanics, mathematical analysis, machine learning, etc. In this paper, we propose a new method for solving the inverse XDE (ODE, PDE, SDE) problems by utilizing the entropy balance equation instead of the original differential equations. This distinguishing feature constitutes a major difference between our current method and other previous classical methods (e.g. SINDy). Despite concerns about the potential information loss during the compression procedure from the original XDEs to single entropy balance equation, various examples from MM reactions, Schlögl model and chemical Lorenz equations in the form of ODEs to nonlinear porous medium equation and Fokker–Planck equation with a double-well potential in the PDE form all well confirm the accuracy, robustness and reliability of our method, as well as its comparable performance with respect to other state-of-the-art algorithms.

Geometric Quasi-Linearization (GQL) for Bound-Preserving Schemes

Kailiang Wu(吴开亮)
Southern University of Science and Technology

Solutions to many partial differential equations satisfy certain bounds or constraints. For example, the density and pressure are positive for equations of fluid dynamics, and in the relativistic case the fluid velocity is upper bounded by the speed of light, etc. As widely realized, it is crucial to develop bound-preserving numerical methods that preserve such intrinsic constraints. Exploring provably bound-preserving schemes has attracted much attention and has been actively studied in recent years. This is however still a challenging task for many systems especially those involving nonlinear constraints. Based on some key insights from geometry, we systematically propose a novel and general framework, referred to as geometric quasilinearization (GQL), which paves a new effective way for studying bound-preserving problems with nonlinear constraints. The essential idea of GQL is to equivalently transform all nonlinear constraints to linear ones, through properly introducing some free auxiliary variables. We establish the fundamental principle and general theory of GQL via the geometric properties of convex regions and propose three simple effective methods for constructing GQL. We apply the GQL approach to a variety of partial differential equations and demonstrate its effectiveness and advantages for studying bound-preserving schemes, by diverse challenging examples and applications which cannot be easily handled by direct or traditional approaches.

Arbitrary Lagrangian-Eulerian finite element methods for two-phase flow with applications

Quan Zhao(赵泉)
University of Science and Technology of China

In this talk, we will analyze numerical approximations for two-phase incompressible flow in the Arbitrary Lagrangian-Eulerian framwork. We consider ALE weak formulation for the bulk equations in both conservative and nonconservative form, which is then coupled to a parametric

formulation for the evolving fluid interface. In order to satisfy the volume preservation, we employ a time-weighted approximation of the interface normal in the parametric formulation together with a suitable discrete pressure space. We also show that novel treaments of the inerita term are curical to the stability of fluid kinetic energy independent of the ALE mesh velocity. Numerical examples and applications are presented to demonstrate the robustness of the introduced methods.

Asymmetric Transport computations in Dirac Models of Topological insulators

Zhongjian Wang(王中剑)
Nanyang Technological University

In this talk we will present a fast and accurate algorithm for computing transport properties of two-dimensional Dirac operators with linear domain walls, which model the macroscopic behavior of the robust and asymmetric transport observed at an interface separating two two-dimensional topological insulators. Our method is based on reformulating the partial differential equation as a corresponding volume integral equation, which we solve via a spectral discretization scheme.

We demonstrate the accuracy of our method by confirming the quantization of an appropriate interface conductivity modeling transport asymmetry along the interface, and moreover, confirm that this quantity is immune to local perturbations. We also compute the far-field scattering matrix generated by such perturbations and verify that while asymmetric transport is topologically protected the absence of back-scattering is not.

Projected Sobolev gradient flows for computing ground state of ultracold dipolar fermi gas

Hanquan Wang(王汉权)
Yunnan University of Finance and Economics

Based on density functional theory (DFT), the ground state for the ultracold fermi gas system with dipole-dipole interaction is a functional minimization problem. We extend the recent work on Sobolev gradient flow for the Gross-Pitaevskii eigenvalue problem, and present continuous projected Sobolev gradient flows for computing the DFT-based ground state solution of ultracold dipolar fermi gas. We prove that the gradient flows have the properties of orthonormal preserving and energy diminishing, which is desirable for the computation of the ground state solution. Many numerical technique for partial differential equation can be used to discretize the time-dependant projected Sobolev gradient flows, which may be an advantage of the method. We propose an efficient and accurate numerical scheme –semi-implicit Euler method in time and Fourier spectral method in space for discretizing these projected Sobolev gradient flows and use them to find the ground states of the fermi gas numerically. Extensive numerical examples in three dimensions for ground states are reported to demonstrate the power of the numerical methods and to discuss the physics of dipolar fermi gas. This is a joint work with Xuelin Zhang.

Inverse wave-number-dependent source problems for the Helmholtz equation

Guanghui Hu(胡广辉)
Nankai University

We consider a multi-frequency factorization method for imaging the support of a wave-number-dependent source function. It is supposed that the source function is given by the Fourier transform of some time-dependent source with a priori given radiating period. Using the multi-frequency far-field data at a fixed observation direction, we provide a computational criterion for characterizing the smallest strip containing the support and perpendicular to the observation direction. The far-field data from sparse observation directions can be used to recover a Θ-convex polygon of the support. The inversion algorithm is proven valid even with multi-frequency near-field data in three dimensions. The connections to time-dependent inverse source problems are discussed in the near-field case. Finally, we show numerical tests and analysis in both two and three dimensions.

Normalized DNN for ground states of BEC

Xiaofei Zhao(赵晓飞)
Wuhan University

The talk is to consider the ground state solution of Bose-Einstein condensates (BEC) by using the deep neural network approach. We first reivew the classical methods and the existing AI methods for the model. Then, a normalized deep neural network is introduced to meet the mass constraint, which turns the constraint minization problem in functional space into a unconstraint minization in parametric space. The approximations of the ground state and also the first excited state can be obtained effectively in a unsupersived learning manner.

Discontinuous Galerkin methods with generalized numerical fluxes for several time dependent convection-dominated PDEs

Xiong Meng(孟雄)
Harbin Institute of Technology

In this talk, we first consider the discontinuous Galerkin method using generalized numerical fluxes for linearized KdV equations. We are able to choose a downwind-biased flux in possession of the anti-dissipation property for the convection term to compensate the numerical dissipation of the dispersion term. This is beneficial to obtain a lower growth of the error and to accurately capture the exact solution without phase errors for long time simulations. By establishing relationships of different numerical viscosity coefficients, a uniform stability is shown. Moreover, a suitable numerical initial condition is chosen. By using generalized Gauss-Radau projections, optimal error estimates are derived. Extensions of discontinuous Galerkin methods with generalized fluxes for nonlinear convection-diffusion systems, nonlinear Schrödinger equations and nonlinear hyperbolic conservation laws are also given. Numerical experiments are provided to confirm the theoretical results.

Quantum simulation of partial differential equations via Schrödingerization

Yue Yu(余跃)
Shanghai Jiao Tong University

We present a simple new way–called Schrödingerization– to simulate general linear partial differential equations via quantum simulation. The idea is to introduce a simple warped

phase transformation that can translate the PDEs into a system of Schrödinger's equations or Hamiltonian system without employing more sophisticated methods. This approach was applied to several typical examples, including the heat, convection, Fokker-Planck, linear Boltzmann and Black-Scholes equations. It can be extended to general linear partial differential equations, including the Vlasov-Fokker-Planck equation and the Liouville representation equation for nonlinear ordinary differential equations. It has the potential to find a variety of applications in time-dependent or independent boundary value problems, artificial and physical and interface boundary conditions.

The singularity swapping method for boundary integral equations and its application to wave scattering

Jun Lai(赖俊)
Zhejiang University

Accurate evaluation of nearly singular integrals plays an important role in many boundary integral equation based numerical methods. In this talk, we propose a variant of singularity swapping method to accurately evaluate the layer potentials for arbitrarily close targets. Our method is based on the global trapezoidal rule and trigonometric interpolation, resulting in an explicit quadrature formula. The method achieves spectral accuracy for nearly singular integrals on closed analytic curves. Numerical examples for Laplace's and Helmholtz equations show that high order accuracy can be achieved for arbitrarily close field evaluation. Extension to three dimensional singular integrals will also be discussed.

Error estimates of numerical methods for the biharmonic nonlinear Schrödinger equation

Ying Ma(马颖)
Beijing University of Technology

The biharmonic nonlinear Schrödinger equation was introduced by Karpman since 1990's and Shagalov, which takes into account the role played by higher-order dispersion terms in the propagation and formation of solitary waves as well as intense laser beams in a bulk medium with Kerr nonlinearity. In this talk, we present a time splitting sine pseudospectral method and two finite difference methods including a Crank-Nicolson scheme and a semi-implicit scheme for the biharmonic nonlinear Schrödinger equation with rigorous error estimates. A time-splitting spectral scheme has the second order convergence rate in temporal and the spectral convergence rate in space. And two finite difference schemes have second order convergence rate both in time and space. Rigorous error estimates of a time-splitting spectral method and two finite difference methods are proved. Several numerical results are reported to confirm our error estimates.

Computation and analysis for action ground states of nonlinear Schrödinger equations

Wei Liu(刘伟)
National University of Singapore

In this talk, I will present some recent results on the numerical computation and mathematical analysis for the action ground states of the nonlinear Schrödinger equation with possible rotation. Here the action ground state refers to a special nontrivial stationary state solution under a

prescribed frequency (or chemical potential) with the minimum action functional value. First, we prove that in focusing regime, the action ground state can be obtained by minimizing a quadratic functional under the L^p-normalization constraint, while in defocusing regime, it can be obtained by the directly minimizing the action functional without constraints. Based on these variational characterizations, we develop efficient gradient flow methods and preconditioned optimization methods to numerically compute the action ground states. Next, we study some mathematical properties of the action ground states in the defocusing case, paying main attention to their connection with the energy ground states. Theoretical equivalence and non-equivalence results and some asymptotic results are established. Numerical evidence for non-equivalence is observed and numerical explorations for vortices phenomena in action ground states are done.

A three-layer Hele-Shaw problem driven by a sink

Meng Zhao(赵蒙)
Huazhong University of Science and Technology

We investigate a sink-driven three-layer flow in a radial Hele-Shaw cell performing numerical simulations. The three fluids are of different viscosities with one fluid occupying an annulus-like domain, forming two interfaces with the other two fluids. Using a boundary integral method and a semi-implicit time stepping scheme, we alleviate the numerical stiffness in updating the interfaces and achieve spectral accuracy in space. The interaction between the two interfaces introduces novel dynamics leading to rich pattern formation phenomena, manifested by two typical events: either one of the two interfaces reaches the sink faster than the other (cusp-like morphology) or they touch each other (interface merging). In particular, the inner interface can be wrapped by the other to have both scenarios. We find that multiple parameters contribute to the dynamics including the width of annular region, the location of the sink, and the mobilities of the fluids.

Fifth-Order A-WENO Schemes Based on the Path-Conservative Central-Upwind Method

Shaoshuai Chu(楚少帅)
Southern University of Science and Technology

We develop fifth-order A-WENO finite-difference schemes based on the path-conservative central-upwind method for nonconservative one- and two-dimensional hyperbolic systems of nonlinear PDEs. The main challenges in development of accurate and robust numerical methods for the studied systems come from the presence of nonconservative products. Semi-discrete second-order finite-volume path-conservative central-upwind (PCCU) schemes recently proposed in Castro Díaz et al. (2019) provide one with a reliable Riemann-problem-solver-free numerical method for nonconservative hyperbolic system. In this paper, we extend the PCCU schemes to the fifth-order of accuracy in the framework of A-WENO finite-difference schemes. We apply the developed schemes to the two-layer shallow water equations, multifluid systems and several traffic flow models. We illustrate the performance of the new fifth-order schemes on a number of one- and two-dimensional examples, where one can clearly see that the proposed fifth-order schemes clearly outperform their second-order counterparts.

A Filon-Clenshaw-Curtis-Smolyak Rule for Multi-Dimensional Oscillatory Integrals

Zhiwen Zhang(张智文)
The University of Hong Kong

We combine the Smolyak technique for multi-dimensional interpolation with the Filon-Clenshaw-Curtis (FCC) rule for one-dimensional oscillatory integration, to obtain a new Filon-Clenshaw-Curtis-Smolyak (FCCS) rule for oscillatory integrals with linear phase over the d-dimensional cube. By combining stability and convergence estimates for the FCC rule with error estimates for the Smolyak interpolation operator, we obtain an error estimate for the FCCS rule. Numerical examples illustrate the theoretical properties of the FCCS rule and include an example of a dimension-adaptive version of our method and an application of our method to an uncertainty quantification (UQ) problem for the Helmholtz equation.

Difference finite element method for 3D incompressible N-S and MHD equations

Xinlong Feng(冯新龙)
Xinjiang University

In this work, a difference finite element (DFE) method is presented for 3D incompressible Navier-Stokes(N-S) and MHD equations. This new method consists of transmitting the FE solution of 3D stead y N-S and MHD equations into a series of the FE solution of 2D steady Oseen iterative equations, which are solved by using the FE pair (P1b,P1b,P1)×P1 satisfying the discrete LBB condition in 2D domain. Moreover, we use FE pair ((P1b,P1b,P1)×P1)×(P1×P0) to solve 3D Oseen iterative equations, where the pair satisfies the discrete LBB condition in 3D domain. Next, we provide the existence and uniqueness of the DFE solutions of 3D Oseen iterative equations and deduce the first order convergence of the DFE solutions to the exact solution of 3D problems. At last, numerical examples are presented to show the accuracy and effectiveness of the proposed method.

Temporal difference learning for high-dimensional PIDEs with jumps

Yi Zhu(朱毅)
Tsinghua University

In this talk, I will introduce a deep learning framework, which we proposed recently, for solving high-dimensional partial integro-differential equations (PIDEs) based on the temporal difference learning. We introduce a set of Levy processes and construct a corresponding reinforcement learning model. To simulate the entire process, we use deep neural networks to represent the solutions and non-local terms of the equations. Subsequently, we train the networks using the temporal difference error, termination condition, and properties of the non-local terms as the loss function. The relative error of the method reaches O(10E-3) in 100-dimensional experiments and O(10E-4) in one-dimensional pure jump problems. Additionally, our method demonstrates the advantages of low computational cost and robustness, making it well-suited for addressing problems with different forms and intensities of jumps.

This is a joint work with Liwei Lu (Tsinghua University), Hailong Guo (Melbourne University) and Xu Yang (UCSB).