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**Title:** Integrable boundary conditions and multidimensional consistency

**Abstract:** Originally discovered by analysing multisoliton solutions of the vector nonlinear Schrödinger equation on the half-line, the so-called set-theoretical reflection equation has quickly found applications to integrable lattice equations on quad-graphs, similarly to the set-theoretical Yang-Baxter equation via its relation to the notion of 3D consistency condition. However, the counterpart of the set-theoretical reflection equation naturally lives on a rhombic dodecahedron and gives rise to the 3D boundary consistency condition. Its interpretation is quite clear as a tool to incorporate integrable boundary conditions for lattice equations on quad-graphs. We will review the results obtained so far in this theory using examples from the Adler-Bobenko-Suris list to illustrate the construction. Time permitting, we will discuss the possibility of combining this theory with the Lagrangian multiform theory of Lobb and Nijhoff.

Xiangke Chang  
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**Title:** On peakon, Toda lattices and associated orthogonal polynomials

**Abstract:** A class of nonlinear integrable PDEs admit some special weak solutions called “peakons”, which are characterised by ODE systems, namely peakon lattices. The celebrated Toda lattice was originally obtained as a simple model for describing a chain of particles with nearest neighbor exponential interaction. For some initial value problems, these lattices can be explicitly solved by use of inverse spectral method involving certain continued fractions, which also associate with some “orthogonality”. In this talk, I will take Camassa-Holm(CH) peakon & Toda lattices & ordinary orthogonal polynomials (OPs), 2-component modified CH interlacing peakon & Kac-van Moerbeke lattices & symmetric OPs, Novikov peakon & B-Toda lattices & Partial skew OPs, Degasperis-Procesi peakon & C-Toda lattices & Cauchy Bi-OPs, as examples to illustrate these connections. Some of the results comes from my recent works with Xing-Biao Hu, Yi He, Shi-Hao Li, Jacek Szmigielski and Jun-Xiao Zhao.

Adri Olde Daalhuis  
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University of Edinburgh

**Title:** Exponentially-improved asymptotics and the higher-order Stokes phenomenon

**Abstract:** Via the Stirling series for the gamma function we introduce resurgence and hyperasymptotics. This will involve the so-called hyperterminants which incorporate the Stokes phenomenon, but also the recently discovered higher-order Stokes phenomenon. The singularity structure of the corresponding Borel plane is used to explain this new phenomenon.

Asymptotics for solutions of difference equations typically give rise to infinitely many singularities in this Borel plane. This can be circumvented by using inverse factorial series instead of simple Poincaré asymptotic series. We will show that there is no free lunch.

Adam Doliwa  
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**Title:** Desargues maps into a projective line

**Abstract:** Abstract of the talk: Desargues maps provide simple incidence-geometric interpretation to (non-commutative in general) discrete KP system. The defining condition is collinearity of images of vertices of basic simplices of the  $A$ -type root lattice. In trying to find the corresponding interpretation of reductions to Painlevé equations one is forced to consider Desargues maps into a projective line where, however, the defining condition loses its meaning. We propose a solution of this problem in terms of the so-called quadrangular set of points, which plays a prominent role in construction of the multiplication in a division ring underlying the projective line. The structure of the division ring may provide geometric meaning for further reductions of Desargues maps. As an example, we show how the circle-geometry interpretation of the discrete KP equation, considered by Konopelchenko and Schief, fits into the Desargues maps scheme.

Vladimir Ejov  
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**Title:** Explicit equations of affine tube domains with large automorphism groups

**Abstract:** Abstract of the talk. This is a joint result with A. Medvedev (SISSA) and G. Schmalz (UNE). We find explicit equations describing tube domains in  $\mathbb{C}^{n+1}$  ( $n \geq 1$ ) with affinely homogeneous base of their boundary and with positive definite and also Lorentzian type Levi form under the condition that their affine isotropy has dimension at least  $\frac{(n-2)(n-3)}{2}$ .

Charalambos Evripidou  
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La Trobe University

**Title:** Integrable and non-commutative integrable Lotka-Volterra systems

**Abstract:** We construct and study a class of Lotka-Volterra systems which we prove that they are Liouville and non-commutative integrable. These Lotka-Volterra systems turn out to be reductions of a class of well known Lotka-Volterra systems, whose Liouville integrability was shown by Bogoyavlenskij and Itoh. We define these systems by giving their interaction matrix; it is a Toeplitz matrix where all of its off-diagonal entries are either plus or minus one. We prove the Liouville and non-commutative integrability of these systems by constructing a set of independent first integrals, having the required involutive properties (with respect to the corresponding Poisson bracket). These first integrals fall into two categories. One set consists of polynomial functions which can be obtained by a matricial reformulation of Itoh's combinatorial description. These polynomial first integrals can also be obtained as the traces of a Lax operator of the systems, which we also construct. The other set consists of rational functions which are obtained through a Poisson map from the first integrals of some recently discovered superintegrable Lotka-Volterra systems.

G. Gubbiotti  
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The University of Sydney

**Title:** Classification of four dimensional rational maps from their first integrals

**Abstract:** Abstract: In a recent paper [N. Joshi and C-M. Viallet, Rational maps with invariant surfaces, arXiv:1706.00173 [nlin.SI], 2017] where provided new examples of rational maps possessing two polynomial first integrals. In this talk we address to the problem of the classification of maps with these properties. We show the complete classification of a class of maps containing both the autonomous form of the  $dP_I^{(2)}$  equation and of the  $dP_{II}^{(2)}$  equation. For every map of the classification we find its dual map in the sense of [G.R.W. Quispel, H.W. Capel and J.A.G. Roberts, J. Phys. A: Math. Gen., 38, 3965, 2005]. We discuss the growth properties of these maps and of their dual maps. We exhibit some examples which show that two first integrals are not sufficient to obtain the condition of slow growth.

Joint work with N. Joshi, D.T. Tran and C-M. Viallet.

Rod Halburd  
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**Title:** Exotic integrable systems in characteristic  $p$

**Abstract:** Some natural analogues of differential equations with solutions defined on functions fields over finite fields will be discussed. Analogues in this setting of important special functions such as the exponential, gamma and hypergeometric functions have been discovered by Carlitz, Goss, Thakur and others. We will discuss integrable analogues of certain differential equations in this context and the role played by singularity analysis. The analogues of differential equations in this context are actually discrete and yet they appear to have more in common with classical integrable differential equations than the familiar discrete integrable systems.

Andrew Hone  
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University of Kent, UK & University of New South Wales, Australia

**Title:** Continued fractions and nonlinear recurrences

**Abstract:** Three-term linear recurrences are a central part of the theory of continued fractions, in description of convergents, as well as being a feature of orthogonal polynomials. This talk will consider some examples of nonlinear recurrences associated with continued fractions. In the setting of the real numbers, some new examples of transcendental numbers with an explicit continued fraction expansion will be provided. The case of function fields associated with algebraic curves will also be mentioned, pointing out the connection with certain discrete integrable systems, Hankel determinants and orthogonal polynomials.

Nalini Joshi  
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**Title:** Open questions for integrable systems

**Abstract:** A field moves forward in many ways, often defined by the interests of a few influential individuals and/or how accessible methods in that field might be to newcomers. These approaches often leave open fundamental questions or necessary connections with other areas. In this talk, I will outline many of these questions and connections that I believe are important.

Kenji Kajiwara  
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Kyushu University

**Title:** Log-aesthetic curves in industrial design as a similarity geometric analogue of Euler's elasticae

**Abstract:** The class of plane curves called the Euler's elastic curves (elasticae) is one of the most important geometric objects and serves as a basic model in the elastic theory. It can be characterized (1) as a critical point of the elastic energy where it is given by the square of the Euclidean curvature, (2) as the stationary flow with respect to the isoperimetric deformation of plane curves in the Euclidean geometry governed by the modified KdV equation, which is one of the most typical integrable systems. In this talk, we consider a class of plane curves called the log-aesthetic curves and their generalization which is used in the industrial design. We investigate those curves under the similarity geometry and characterize them as stationary integrable flow on plane curves which is governed by the Burgers equation. We propose a variational formulation of those curves whose Euler-Lagrange equation yields the stationary Burgers equation. Our result suggests that the log-aesthetic curves and their generalization can be regarded as the similarity geometric analogue of the Euler's elastic curves, which provides a new mathematical framework of those curves and would yield various generalizations. Finally, we construct a discreteization of the log-aesthetic curve based on this framework and formulate it in terms of the discrete variational principle.

Andrew Kels  
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University of Tokyo

**Title:** Yang-Baxter/3D-consistency correspondence

**Abstract:** I will show how each 3D-consistent equation in the Adler-Bobenko-Suris (ABS) classification, respectively arises from a more general equation known as the Yang-Baxter equation. The latter Yang-Baxter equation is a condition of integrability for two-dimensional models of statistical mechanics, and provides a natural path integral quantization of a respective ABS equation. I will use specific examples of Yang-Baxter equations that may be interpreted as transformation formulas for hypergeometric integrals. I will also summarise other aspects of this correspondence, including a connection between the Z-invariance property for integrable models of statistical mechanics, and a Lagrangian multiform closure property for systems of classical discrete Laplace ABS equations.

Qingping Liu  
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China University of Mining and Technology (Beijing)

**Title:** A supersymmetric Sawada-Kotera equation

**Abstract:** Sawada-Kotera equation is one of the typical integrable equations and has been studied extensively. In this talk, we introduce a supersymmetric version for the Sawada-Kotera equation and demonstrate that this system is integrable. Also, a Darboux transformation is constructed for the supersymmetric Sawada-Kotera system. This is a joint work with Kai Tian, Hui Mao and Lingling Xue.

References:

- [1] K. Tian and Q. P. Liu, *Phys. Lett. A* **373** (2009) 1807.
- [2] K. Tian and Q. P. Liu, *AIP Conference Proceedings* **1212** (2010) 81.
- [3] K. Tian and J. P. Wang, *Stud. Appl. Math.* **138** (2017) 467-498.
- [4] Hui Mao, Q. P. Liu and Lingling Xue, *J. Nonlinear Math. Phys.* (to appear).

Akane Nakamura  
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**Title:** The Bäcklund transformations of the matrix Painlevé equations

**Abstract:** We consider the Bäcklund transformations of the sixth matrix Painlevé equations induced by the symmetries (a middle convolution, additions, a Schlesinger transformation, etc.) of the linear equation. The Bäcklund transformations of other matrix Painlevé equations can be obtained by considering degeneration processes.

Nobutaka Nakazono  
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Aoyama Gakuin University

**Title:** CAC cube structure of the discrete power function

**Abstract:** In this talk, we show that the discrete power function associated with circle patterns of Schramm type can be obtained from a space-filling cubic lattice, each cube has CAC property, and its affine Weyl group symmetry. Moreover, we show that this cubic lattice and its symmetry are derived from the affine Weyl group symmetry of the sixth Painlevé equation. This work has been done in collaboration with Profs Nalini Joshi, Kenji Kajiwara, Tetsu Masuda and Dr Yang Shi.

Linyu Peng  
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Waseda University

**Title:** A semi-discrete version of Noether's theorem

**Abstract:** This year, we celebrate centenary of the publication of Emmy Noether's paper on invariant variational problems (*"Invariante Variationsprobleme"*). The main result of the paper, namely Noether's theorem, is of fundamental importance in understanding about nature. This theorem states that symmetries of action functionals lead to conservation laws of the equations of motion and has been extended to finite difference, finite element and semi-discrete cases. In this talk, we dis-

cuss a recent construction of its semi-discrete version with a study of symmetry prolongations about semi-discrete systems. Worked examples of integrable semi-discrete equations will be provided.

GRW Quispel  
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**Title:** Kahan's Method and Discrete Integrable Systems

**Abstract:** We discuss Kahan's method for discretizing quadratic ordinary differential equations (ODEs), and some novel integrable maps thus obtained by discretizing integrable ODEs.

John A G Roberts  
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University of New South Wales

**Title:** How many integrals does a map have?

**Abstract:** The title gives one of the basic questions one might pose for a map of  $\mathbb{R}^d$  to itself. The answer could be an integer  $j$  lying between 0 (the generic case, mathematically speaking) and  $d - 1$ , the super-integrable case. We restrict to the case where the map is rational and we search for rational integrals. In this algebraic setting, each orbit is restricted to the algebraic variety of dimension  $d - j$  defined by the intersection of the level sets of  $j$  integrals. We describe a fast diagnostic that infers  $j$  correctly for many rational maps. It is based upon studying the dynamics of the rational map over a finite field. When  $d = 2$ , the inference of  $j = 0$  or  $j = 1$  was developed by Roberts and Vivaldi (2003), using the Hasse-Weil bound. For  $d > 2$ , our diagnostic is based upon other considerations. I will explain the method and indicate possible limitations, complications and extensions. This is joint work with Tim Siu (UNSW).

Hidetaka Sakai  
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University of Tokyo

**Title:** CFT approach to the  $q$ -Painlevé equations

**Abstract:** Iorgov, Lisovyy, and Teschner established a connection between isomonodromic deformation of linear differential equations and Liouville conformal field theory at  $c = 1$ . In this talk we present a  $q$  analog of their construction. We show that the general solution of the  $q$  Painlevé VI equation is a ratio of four tau functions, each of which is given by a combinatorial series arising in the AGT correspondence. This talk is based on a joint work with M. Jimbo and H. Nagoya.

Harvey Segur  
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**Title:** Tsunami

**Abstract:** People around the world have become more interested in tsunami, because of two events: the 2004 tsunami that killed hundreds of thousands of people who lived near the coastline of the Indian Ocean, and the 2011 tsunami off the coast of Japan, which created a nuclear disaster that

continues to endanger people who live near that reactor. This talk will describe the dynamics of a tsunami, what makes them dangerous, and what can be done to protect people from the destructive effects of a tsunami. The dynamics of a tsunami are described with reasonable accuracy by the equations of fluid mechanics, but my objective in this talk is to show how the implications of those equations fit with the observations of the two tsunamis mentioned above.

Yang Shi  
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**Title:** Normalizer of parabolic subgroups in Coxeter groups and symmetry of discrete integrable equations

**Abstract:** We explore some nice properties of the Coxeter groups in context of discrete integrable equations. In particular, we look at the Normalizer of parabolic subgroups in Coxeter groups, which give us a way to explain the occurrences of semidirect products of groups as symmetries for discrete integrable equations.

Yingying Sun  
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**Title:** A study of the continuous and discrete Boussinesq systems

**Abstract:**

It has been long understood how to interpret the permutability formula of the Bäcklund transformation as a lattice equation. I will talk about a recent result showing the lattice Boussinesq equation can be derived from a Bäcklund transformation of the potential Boussinesq system. This Bäcklund transformation is constructed via Weierstrass elliptic functions. I will then report on joint work with Dajun Zhang and Frank Nijhoff how to obtain the elliptic seed and one soliton solution of the lattice Boussinesq equation.

Tomoyuki Takenawa  
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**Title:** The space of initial conditions for some 4D Painlevé systems

**Abstract:** In recent years, research on 4D Painlevé systems have progressed mainly from the viewpoint of isomonodromy deformation of linear equations. In this talk we study the geometric aspects of 4D Painlevé systems by investigating the space of initial conditions in Okamoto-Sakai's sense, which was a powerful tool in the original 2D case. Specifically, starting from known discrete symmetries, we construct the space of initial conditions for some 4D Painlevé systems, and using the Néron-Severi bilattice, clarify the whole group of their discrete symmetries. The examples include the directly coupled 2D Painlevé equations, Noumi-Yamada's  $A_5^{(1)}$  system and the 4D Garnier system.

Dinh Tran  
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**Title:** On a multi-parametric equation and its reductions

**Abstract:** In this talk, I will start with a multi-parametric equation obtained as a "double twist" of  $Q_1$  in the Adler-Bobenko-Suris classification (2003). This equation turns out to be consistent around the cube. We then use special coefficients to derive a non-symmetric equation (Hietarinta-Viallet equation) and obtain a Lax pair for this equation. Periodic reductions will be used to derive several classes of ordinary difference equations such as Lyness' equation, Demskoy-Tran-Kamp-Quispel (DTKQ) equation and Svinin's equations.

Teruhisa Tsuda  
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Hitotsubashi University

**Title:** Rational approximation and Schlesinger transformation

**Abstract:** We show how rational approximation problems for functions are related to the construction of Schlesinger transformations. We then discuss their applications to the theory of isomonodromic deformations or Painleve equations.

References:

- [1] Ishikawa, M., Mano, T., Tsuda, T.: Determinant structure for  $\tau$ -function of holonomic deformation of linear differential equations, preprint.
- [2] Mano, T., Tsuda, T.: Hermite-Pade approximation, isomonodromic deformation and hypergeometric integral, Math. Z. 285 (2017)

Peter Van der Kamp  
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**Title:** Dressing the dressing chain

**Abstract:** The dressing chain arises in the application of Darboux transformations to the Schrödinger equation, which is the spectral problem for the Korteweg-De Vries (KdV) equation. It also arises as an auto-Bäcklund transformation of the modified KdV equation (mKdV) via the celebrated Miura transformation between KdV and mKdV, and a symmetry of mKdV. Odd dimensional periodic reductions of the dressing chain were shown to be completely integrable by Veselov and Shabat. In this talk I will demonstrate that one can replace KdV by dressing chain in the above text:

The lattice KdV equation arises in the application of Darboux transformations to the discrete Schrödinger equation, which is the spectral problem for the dressing chain (DC). It also arises as an auto-Bäcklund transformation of a modified dressing chain (mDC) via a Bäcklund transformation between DC and mDC, and a symmetry of mDC. Odd dimensional periodic reductions of the lattice KdV equation are shown to be completely integrable by Evripidou, Zhang and me.



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**Title:** Inflation of a cubic pencil

**Abstract:** We apply the “inflation” process introduced in a recent work with N. Joshi to a simple integrable map in two dimensional projective space ( $CP_2$ ) possessing an invariant pencil of cubics. We obtain a new birational map acting in three dimensional projective space ( $CP_3$ ). We analyse in some detail the singularity structure of this map as well as the growth rate of the degrees of its iterates, *proving in particular that the growth is cubic*. This, together with the picture of orbits, implies that the map is integrable but leaves invariant *non-algebraic curves drawn on two dimensional algebraic surfaces*, a new phenomenon in the game.

Yasuhiko Yamada  
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Kobe University

**Title:** Comments on  $q$ -Garnier systems

**Abstract:** A  $q$ -difference analog of the Garnier systems was first formulated by H.Sakai. In this talk, I will give several reformulations of the  $q$ -Garnier systems and discuss their geometric structure by considering their autonomous limit. I will also consider the bilinear form for some degenerate cases.

Sikarin Yoo-Kong  
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**Title:** The Hamiltonian Zoo for the harmonic oscillator

**Abstract:** We present alternative forms of the standard Hamiltonian called Newton-equivalent Hamiltonian Zoo, giving the same equation of motion, for the harmonic oscillator. These Hamiltonians are solved directly from the Hamilton’s equations and come with extra-parameters which are interpreted as time scaling factors.

Cheng Zhang  
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**Title:** Darboux-Crum transformations for KdV-type quad-equations

**Abstract:** Darboux transformations and the associated Crum’s theorem are important aspects in integrability. In this talk, I will fit the Darboux-Crum scheme into the KdV-type 3D-consistent quad-equations (the lattice potential, modified potential and Schwarzian KdV equations). This follows from explicit constructions of Darboux-Crum transformations for two second order difference spectral problems, which are themselves discretised versions (using Darboux transformations) of the spectral problems of the KdV and mKdV equations. It turns out that, along the Darboux-discretisation processes, two families of integrable systems (the KdV family, and the modified KdV family), including their continuous, semi-discrete and lattice versions, are explicitly connected. Multi-soliton solutions of those lattice equations can be obtained thanks to the Darboux-Crum transformations. This work is in collaboration with Linyu Peng and Dajun Zhang.

Youjin Zhang  
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**Title:** Fractional Volterra hierarchy and its application

**Abstract:** We construct a new integrable hierarchy in terms of Lax pair and Hamiltonian formalisms, it is parametrized by two complex numbers and is called the fractional Volterra hierarchy. For a special choice of the two complex numbers we obtain the well-known Volterra hierarchy (or the discrete KdV hierarchy). We construct its soliton solutions and show its relationship to the study of cubic Hodge integrals satisfying the local Calabi-Yau condition.