Titles and Abstracts

1. William Beckner, University of Texas at Austin, U.S.

Title: Symmetry in Fourier Analysis

Abstract: Symmetry is instrumental to the framework for understanding differential and integral operators that arise in Fourier analysis. As outlined more than fifty years ago by Eli Stein, and earlier by Weil and Bochner, Lie groups are the central domain for extending the tools of Fourier analysis beyond Euclidean space. Basic Lie groups to study are $\mathrm{SL}(2, R)$, the Heisenberg group and hyperbolic space with the goal of obtaining new analytic insight for Sobolev embedding, Riesz potentials, Stein-Weiss integrals and Kunze-Stein phenomena. Embedded symmetry within the Heisenberg group can be used to couple geometric information and analytic calculation to obtain new inequalities on Euclidean space. The intrinsic character of the Heisenberg group makes it the natural playing field on which to explore the laws of symmetry and the interplay between analysis and geometry on a manifold.

2. Bo Guan, Ohio State University, U.S

Title: Inequalities for convex functions and fully nonlinear elliptic equations

Abstract: Fully nonlinear elliptic and parabolic equations on manifolds play central roles in some important problems in real and complex geometry. A key ingredient in solving such equations is to establish apriori estimates up to second order. For general Riemannian manifolds, or Kaehler/Hermitian manifolds in the complex case, one encounters difficulties caused by the curvature (as well as torsion in the Hermian case) of the manifolds.

In this talk we report some results in our effort to overcome these obstacles over the past few years. We emphasize on understanding the roles of subsolutions and concavity of the equation based on which our techniques were developed and present some related inequalities for concave functions.

We are interested both in equations on closed manifolds, and the Dirichlet problem for equations on manifolds with boundary of arbitrary geometry.

For the Dirichlet problem on manifolds with boundary, we prove that under some fundamental structure conditions which were first proposed by Caffarelli-Nirenberg-Spruck and are now standard in the literature, there exists a smooth solution provided that there is a C2 subsolution.

For equations on closed manifolds, there have appeared two different notations of weak subsolutions,

the C-subsolution introduced by Gabor Szekelyshidi (JDG, 2018) and "tangent cone at infinity" condition by myself (Duke J Math, 2014). We show for type I cones the two notations coincide. We also construct examples showing for the Dirichlet problem that the subsolution condition can not be replaced by the weaker versions.

3. Lu Chen, Beijing Institute of Technology, China

Title: Existence and Nonexistence of Extremals for the Adams Inequalities in $\mathbb{R}^4$.

Abstract: Much work has been done with respect to the existence of extremals of the critical Trudinger-Moser inequalities in unbounded domain. However, whether there exist extremal functions for the critical Adams inequalities in unbounded domain still remains open. The classical blow-up procedure can not apply to solving the existence because of the presence of Pólya-Szegö type inequality. In this paper, we develop some new ideas based on the sharp Fourier rearrangement principle, sharp constants of the higher-order Gagliardo-Nirenberg inequalities and poly-harmonic truncations to study the existence and nonexistence of the maximizers for the Adams inequalities. Our results also provides a further insight on the existence or nonexistence of extremals for Adams inequality which has not been noticed before, even in the case of Trudinger-Moser inequality. Furthermore, the symmetry of the extremal functions can also be deduced through the Fourier rearrangement principle. This the joint work with Prof. Lu and Prof. Zhu.

4. Ting Chen, Nankai University, China

Title: Fractional Integrals on Lebesgue Spaces with Mixed Norms

Abstract: Lebesgue spaces with mixed norms were first studied by Benedek and Panzone in 1961, where many fundamental properties were proved. In particular, they showed that such spaces possess similar properties as usual Lebesgue spaces. In this talk, we focus on the fractional integral operators on mixed Lebesgue spaces.

5. Qianqiao Guo, Northwestern Polytechnical University, China

Title: Subcritical Approach to Sharp Hardy-Littlewood-Sobolev Type Inequalities on the Upper Half Space

Abstract: We establish the reversed sharp Hardy-Littlewood-Sobolev (HLS for short) inequality on the upper half space and obtain a new HLS type integral inequality on the upper half space (extending an inequality found by Hang, Wang and Yan) by introducing a uniform approach. The extremal functions are classified via the method of moving spheres, and the best constants are computed. The new approach can also be applied to obtain the classical HLS inequality and other similar inequalities. This is a joint work with Prof. Jingbo Dou and Prof. Meijun Zhu.

6. Yuxia Guo, Tsinghua Univeristy, China

Title: Infinitely Many Solutions for Fractional Schrödinger Equation Involving Critical Exponent

Abstract: We consider the following fractional Schrödinger equation involving critical exponent:

$$\begin{cases} (-\Delta)^s u + V(y)u = u^{2_s^*-1} \ \text{ in } \ \mathbb{R}^N, \\ y \in \mathbb{R}^N, \end{cases} \tag{P}$$

where $N \geq 3$, $2_s^* = \frac{2N}{N-2s}$ is the critical Sobolev exponent, $V(y)$ is a bounded nonnegative function with a weaker symmetry condition. Under some suitable assumptions of the potential function $V(y)$, by using a finite dimensional reduction method combining with various local Pohazaev identities, we prove the existence of infinitely many solutions for the problem

(P). Due to the non-localness of the fractional Laplacian operator, we need to study the corresponding harmonic extension problem.

7. Pak Tung Ho, Sogang University, Korea.

Title: Uniqueness Result on CR Manifolds

Abstract: Bidaut-Veron and Veron showed the uniqueness of the constant solution to a general Yamabe-type equation on a compact Riemannian manifold. I will explain the idea of their proof and some applications. I will then explain how one can generalize their result to CR manifold.

8. Nguyen Lam, University of British Columbia, Canada.

Title: Best Constants and Optimizers for Sobolev Type Inequalities with Weights

Abstract: Sharp geometric and functional inequalities play an important role in applications to geometry and PDEs. In this talk, we will describe some recent works on best constants and optimizers of several Sobolev type inequalities with the presence of the weights. Best constants are identified and existence of extremizers are proved, and in many cases explicit forms of extremal functions are found.

9. Yuxiang Li, Tsinghua University, China.

Title: A Collapsing Sequence in a Conformal Class with Bounded Volume and $\|\mathrm{Ric}\|_{L^p}$

Abstract: We will use Moser-Trudinger inequality to study a collapsing sequence in a fixed conformal class under some curvature conditions. As an application, we will show that when $\dim M > 2$, a metric sequence g_k in a conformal class with $\mathrm{vol}(g_k) + \|\mathrm{Ric}(g_k)\|_{L^p} < C$ for some $p > \frac{n}{2}$, converges in $C^{1,\alpha}$.

10. Guozhen Lu, University of Connecticut, U.S.

Title: Fourier analysis on hyperbolic spaces and Hardy-Sobolev-Maz'ya and Hardy-Trudinger-Moser-Adams inequalities on upper half spaces

Abstract: Sharp geometric inequalities play an important role in analysis and differential geometry. In this talk, we will review some recent works on sharp Hardy-Sobolev-Maz'ya inequalities on the upper half space which improve the classical Sobolev inequality. We will also discuss the borderline case of the Sobolev inequalities, namely, the Trudinger-Moser and Adams inequalities on hyperbolic spaces. In particular, we will describe the Fourier analysis techniques on the hyperbolic spaces and their applications to sharp geometric inequalities.

11. Gabriele Mancini, Università Sapienza di Roma, Italy

Title: Bubbling Nodal Solutions for a Large Perturbation of the Moser-Trudinger Equation on Planar Domains

Abstract: I will discuss some results obtained in collaboration with Massimo Grossi, Angela Pistoia and Daisuke Naimen concerning the existence of nodal solutions for the problem

$$-\Delta u = \lambda u e^{u^2+|u|^p} \quad \text{in } \Omega, \quad u = 0 \quad \text{on} \quad \partial\Omega$$

where $\Omega \subseteq \mathbb{R}^2$ is a bounded smooth domain and $p \to 1^+$. If Ω is ball, it is known that the case $p = 1$ defines a critical threshold between the existence and the non-existence of radially symmetric sign-changing solutions with λ close to 0. In our work we construct a blowing-up family of nodal solutions to such problem as $p \to 1^+$, when Ω is an arbitrary domain and λ is small enough. To our knowledge this is the first construction of bubbling sign-changing solutions for a Moser- Trudinger type critical equation on a non-symmetric domain.

12. Luca Martinazzi, Università degli Studi di Padova, Italy

Title: News on the Moser-Trudinger Inequality: From Sharp Estimates to the Leray-Schauder Degree

Abstract: The existence of critical points for the Moser-Trudinger inequality for large energies has been open for a long time. We will first show how a collaboration with G. Mancini allows to recast the Moser-Trudinger inequality and the existence of its extremals (originally due to L. Carleson and A. Chang) under a new light, based on sharp energy estimate. Building upon a recent subtle work of O. Druet and P-D. Thizy, in a work in progress with O. Druet, A. Malchiodi and P-D. Thizy, we use these estimates to compute the Leray-Schauder degree of the Moser-Trudinger equation (via a suitable use of the Poincaré-Hopf theorem), hence proving that for any bounded non-simply connected domain the Moser-Trudinger inequality admits critical points of arbitrarily high energy. In a work in progress with F. De Marchis, O. Druet, A. Malchiodi and P-D. Thizy, we expect to use a variational argument to treat the case of a closed surface.

13. Kunnath Sandeep, TIFR Centre for Applicable Mathematics, India.

Title: Adams Inequalities in Hadamard Manifolds

Abstract: Let Ω be a bounded domain in $\mathbb{R}^n$, then the classical Sobolev embedding states that $W^{k,p}(\Omega) \hookrightarrow L^q(\Omega)$ for all $q < \infty$ when $kp = n$. The Moser-Trudinger and Adams inequalities describe a sharp version of the embedding of $W^{k,p}(\Omega)$ when $kp = n$. This inequality has been studied in various contexts like compact Riemannian manifolds. In this talk we describe some results in the case of noncompact setting.

14. Weimin Sheng, Zhejiang University, China.

Title: An Anisotropic Shrinking Flow and L_p Minkowski Problem

Abstract: In this talk, I will introduce my recent work with Caihong Yi on studying anisotropic shrinking flows and the application on L_p Minkowski problem. We consider an shrinking flow of smooth, closed, uniformly convex hypersurfaces in Euclidean $\mathbb{R}^{n+1}$ with speed $fu^{\alpha}\sigma_n^{-\beta}$, where u is the support function of the hypersurface, $\alpha, \beta \in \mathbb{R}^1$, and $\beta > 0$, σ_n is the n-th symmetric polynomial of the principle curvature radii of the hypersurface. We prove that the flow exists an unique smooth solution for all time and converges smoothly after normalisation to a smooth solution of the equation $fu^{\alpha-1}\sigma_n^{-\beta} = c$ provided the initial hypersurface is origin-symmetric and f is a smooth positive even function on S^n for some cases of α and β. In the case $\alpha \geq 1 + n\beta$, $\beta > 0$, we prove that the ow converges smoothly after normalisation to a unique smooth solution of $fu^{\alpha-1}\sigma_n^{-\beta} = c$ without any constraint

on the initial hypersurface and the function f. When $\beta = 1$, our argument provides a uniform proof to the existence of the solutions to the L_p Minkowski problem $u^{1-p}\sigma_n = \phi$ for $p \in (-n-1, +\infty)$, where ϕ is a smooth positive function on S^n.

15. Wenchang Sun, Nankai University, China.

Title: Bloom Type Inequality: The Off-Diagonal Case

Abstract: In this talk, we give a representation formula for fractional integrals. As a consequence, we get a Bloom type inequality for the Ferguson-Lacey type commutator involved with fractional integrals. Our results extend similar results for singular integral operators. The main difference is that mixed-norm spaces are invoked when we study the off-diagonal case of Bloom type inequalities for fractional integrals.

16. Zhongwei Tang, Beijing Normal University, China

Title: Solutions for Conformally Invariant Fractional Laplacian Equations with Multi-bumps Centered in Lattices

Abstract: In this talk, we consider the following nonlinear elliptic equation involving the fractional Laplacian with critical exponent:

$$(-\Delta)^s u = K(x)u^{\frac{N+2s}{N-2s}},\ u > 0 \text{ in } \mathbb{R}^N,$$

where $s \in (0,1)$ and $N > 2+2s$, $K > 0$ is periodic in $(x_1, \ldots, x_k)$ with $1 \le k < \frac{N-2s}{2}$. Under some natural conditions on K near a critical point, we prove the existence of multi-bump solutions where the centers of bumps can be placed in some lattices in $\mathbb{R}^k$, including infinite lattices. On the other hand, to obtain positive solution with infinite bumps such that the bumps locate in lattices in $\mathbb{R}^k$, the restriction on $1 \le k < \frac{N-2s}{2}$ is in some sense optimal, since we can show that for $k \ge \frac{N-2s}{2}$, no such solutions exist. This is a joint work with Dr. Miaomiao Niu and Dr.Lushun Wang.where $s \in (0,1)$ and $N > 2+2s$, $K > 0$ is periodic in $(x_1, \ldots, x_k)$ with $1 \le k < \frac{N-2s}{2}$. Under some natural conditions on K near a critical point, we prove the existence of multi-bump solutions where the centers of bumps can be placed in some lattices in $\mathbb{R}^k$, including infinite lattices. On the other hand, to obtain positive solution with infinite bumps such that the bumps locate in lattices in $\mathbb{R}^k$, the restriction on $1 \le k < \frac{N-2s}{2}$ is in some sense optimal, since we can show that for $k \ge \frac{N-2s}{2}$, no such solutions exist. This is a joint work with Dr. Miaomiao Niu and Dr. Lushun Wang.

17. Neil Trudinger, Australian National University, Canberra, Australia

Title: REMARKS ON CURVATURE EQUATIONS, HESSIAN EQUATIONS AND ISOPERIMETRIC INEQUALITIES

Abstract: We clarify some earlier results from the late 1980s and early 1990s pertaining to the relationship between curvature equations, Hessian equations and isoperimetric inequalities. These investigations arose from an interest in obtaining sharp height estimates for graphs with prescribed curvature, extending those of Giaquinta-Giusti for mean curvature and Bakel'man for Gauss curvature, and led naturally to the issue of extensions of the Alexandrov-Fenchel isoperimetric inequalities for quermassintegrals to non-convex domains.

In particular we indicate an approach to sharp volume inequalities using a perturbed Gauss curvature equation in arbitrary domains.

18. Zhi-Qiang Wang, Utah State University, U.S.

Title: Borderline Asymptotics in Sobolev Embedding

Abstract: We report recent work on the asymptotic behavior of extremal functions for the Sobolev embedding from H^1 into L^p as $p \to 2$. This gives a precise convergence result of the ground states of the scalar field equation, and in turn yields a direct proof of the logarithmic Sobolev inequality by passing limit in the Sobolev embedding. This is a joint work with Chengxiang Zhang.

19. Ruijun Wu, Max Planck Institute for Mathematics in the Sciences, Germany.

Title: Blow Up Analysis and Energy Identity for a Nonlinear Sigma Model

Abstract: We will take about a model from physics which is a natural extension of the harmonic maps. The maps and Riemannian metrics obtains their super partners given by suitable spinorial fields. We will give a description of the the compactness of the solutions with bounded energies and with an algebraic constraint. The bubbles are given by Dirac harmonic maps with curvature terms and the energies are preserved along the blowup. This is a joint work with J. Jost and M. Zhu.

20. Qiaohua Yang, Wuhan University, China

Title: An Extension Problem and Trace Sobolev Inequalities

Abstract: In this talk, I will discuss the connection between the extension problem and the trace Sobolev inequality. By using, among other things, the scattering theory on hyperbolic spaces (including ball model and half space model) and the generalized Poisson kernel, we derive the sharp trace Sobolev inequalities on half spaces and balls for higher order derivatives and obtain the explicit formulas of extremal functions of such inequalities, which generalize the result of Ache and S.-Y. A. Chang et al.

21. Dong Ye, East China Normal University, China

Title: Get Hardy-Rellich Inequalities via Equalities

Abstract: We try to get Hardy-Rellich type inequalities using exact formulae, inspired by the recent work of Machiara-Ozawa-Wadade. We find many classical Hardy-Rellich type inequalities with best constants and simple proofs, moreover these equalities mean readily that all the best constants cannot be reached. This is a joint work with Xia Huang at ECNU.

22. Lu Zhang, Binghamton University, U.S.

Title: Trudinger-Moser inequalities under different norms and their extremal functions

Abstract: Most of the earlier works on Trudinger-Moser inequalities were established based on the symmetrization method, and the existence of their extremal functions were studied via the complicated blow-up analysis. We establish a new approach, which can successfully

overcome some of the limitations of the earlier methods, to study such inequalities. More precisely, we are able to study subcritical, critical singular Trudinger-Moser inequalities on $\mathbb{R}^n$, prove their equivalence, and establish these inequalities under different norms. Moreover, with the equivalence and the relation between the suprema of the subcritical and critical inequalities, we can have a different view of looking at the extremal functions of these inequalities, where many new properties can be obtained. (Joint work with N. Lam and G. Lu).

23. Feng Zhou, East China Normal University, China

Title: On Isolated Singular Solutions to Lane-Emden Equations

Abstract: In this talk I will discuss isolated singular solutions to Lane-Emden equation $-\Delta u = Vu^p$ in $R^N \setminus \{0\}$, where $p > 1$, V is a nonhomogeneous potential satisfying some extra hypotheses. We construct a sequence of fast decaying solutions and slow decaying solutions. This is based on joint work with HY. Chen and X.Huang.

24. Maochun Zhu, Jiangsu University, China

Title: Existence of Ground State Solutions for Bi-harmonic Equations with Exponential Critical Growth Involving Different Potentials in $\mathbb{R}^4$

Abstract: In this talk we will present some new results concerning the existence of ground state solutions for bi-harmonic equations involving constant and general non-constant potentials. Firstly, we prove the existence of ground state solutions of bi-harmonic equations with constant potentials:

$$\Delta^2 u + u = \gamma u \exp(2u^2) \text{ in } \mathbb{R}^4,$$

where $0 < \gamma < 1$. Since the nonlinearity does not satisfy the Ambrosetti-Rabinowitz condition, which make it impossible to use the mountain-pass theorem. To overcome this, we exploit the necessary and sufficient conditions for the boundedness and the compactness of general nonlinear functional in $H^2(\mathbb{R}^4)$; secondly, we prove the existence of ground state solutions of bi-harmonic equations with general non-constant potentials,

$$\Delta^2 u + V(x)u = \gamma u \exp(2u^2) \text{ in } \mathbb{R}^4,$$

where γ and the potential $V(x)$ satisfy

$$0 < \gamma < \inf_{x\in\mathbb{R}^4} V(x) < \lim_{|x|\to\infty} V(x) = 1,$$

for this, we will apply the Nehari manifold methods. This part of the talk is based on joint work with Guozhen Lu and Lu Chen.