



丘成桐数学科学中心  
YAU MATHEMATICAL SCIENCES CENTER



# 2023

## 红外同调代数研讨会

HOMOLOGICAL ALGEBRA OF THE INFRARED

会议手册 HANDBOOK

2023.1.8-14

清华大学丘成桐数学科学中心 (YMSC)  
清华三亚国际数学论坛 (TSIMF)

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# Prof. S. -T. Yau



Chair Professor at Tsinghua University, Director of Yau Mathematical Sciences Center at Tsinghua University, Dean of Qiuzhen College at Tsinghua University, President of BIMSA, Professor at Harvard University, Member of the United States National Academy of Sciences, Member of American Academy of Arts and Sciences, Foreign Member of the Chinese Academy of Sciences, Winner of Fields Medal, Crafoord Prize, Wolf Prize and Marcel Grossmann Award.

## Prizes and Awards

- 1981 Oswald Veblen Prize
- 1982 Fields Medal
- 1984 MacArthur Fellow
- 1994 Crafoord Prize
- 1997 United States National Medal of Science
- 2003 China International Scientific and Technological Cooperation Award
- 2010 Wolf Prize in Mathematics
- 2018 Grossmann Marcell Prize

# About YMSC

In December 2009, Tsinghua University established Mathematical Sciences Center, with internationally renowned mathematics master Professor Shing-Tung Yau appointed as the Director of the center.

As a major strategic measure to support development in mathematics at Tsinghua University, the Ministry of Education officially approved the establishment of Yau Mathematical Sciences Center managed by Tsinghua University at the end of 2014.

Under the leadership of Professor Shing-Tung Yau, YMSC has made substantial development in top talents recruitment, outstanding mathematical talents cultivation, cutting-edge scientific research and mathematical discipline construction, making it a world-class research center of significant international influence.



As the important base for talents development and academic research of basic sciences in China, YMSC plays an important role for the development of Tsinghua as first-class university. Following the strategy of building an “international, open and academic” institute, YMSC has attracted a large number of top mathematical scholars, cultivated a group of outstanding young talents, and formed a comprehensive academic layout comprised of “five major research areas” and “three cross-disciplinary research directions”. YMSC prides itself on favorable environment for talent development and student cultivation supported by its world-renowned faculty, first-class international communication platform, as well as open and active academic atmosphere.

YMSC treats faculty development and students cultivation as its fundamental tasks. In the past years, YMSC has attracted 86 full-time faculty members with strong support from Tsinghua University, including 26 professors, 12 associate professors and 48 assistant professors. Currently, YMSC has 49 postdoctoral researchers.



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Around faculty development and student cultivation, YMSC keeps exploring and innovating the modes of mathematical talents selection and cultivation. It has established a series of talent selection and communication platforms including "S. T. Yau High School Science Award", "S.-T. Yau College student Mathematics Contests", "ICCM Best Thesis Award", "ICCM Medal of Mathematics", "Distinguished Lectures", "Master Forums", "International Chinese Mathematical Union" and "International Congress of Chinese Mathematicians". In addition, it has signed Memoranda of Understanding with relevant departments of nearly 30 world top universities and research institutes, such as Harvard University, Stanford University and Oxford University. More than 300 mathematicians as well as experts and scholars in related areas visit YMSC each year for exchange and communication. It has also hosted many important international academic conferences at Tsinghua University or Tsinghua Sanya International Mathematical Forum. A large number of academicians in various countries as well as winners of the Nobel Prize, the Fields Medal and Wolf Prize have visited YMSC for communication and speech delivery.

As mathematical sciences are facing an unprecedented historical opportunity, YMSC will uphold the development principle of exploitation, innovation and long-term development, serve the national strategic needs, aim at world's science and technology frontier, encourage both faculty members and students to make concerted efforts, continue to make breakthroughs in first-class innovative talents cultivation, international original scientific research, mathematical discipline construction and faculty development. YMSC will endeavor to build a first-class mathematics center and a first-class discipline in the world.

# About TSMIF

The Tsinghua Sanya International Mathematics Forum (TSMIF) was initiated the proposal by Shing-Tung Yau, an internationally renowned mathematics master. With the support of the Central Government and the strong support of Hainan Province and Sanya City, an international academic conference center was built by Tsinghua University.



TSIMF is devoted to providing an elegant and quiet exchange environment for academic research. It has developed into a base of scientific innovation through academic exchanges and cooperation among the world's leading purely math, applied mathematics, statistics, theoretical physics, applied physics and other disciplines research teams. It will provide a platform for exploring new research directions, developing new theories, cultivating outstanding math talents and raising the level of Chinese mathematics research. TSIMF is intended to become a world-class international conference center holding workshops and international conferences in mathematics all year long, and it is the first of this type in Asia.

TSIMF is located in the southernmost tropical coastal city of Hainan Province, Sanya City. It is also located in the northwestern Phoenix Ridge mountains, adjacent to Sanya Phoenix International Airport. It is east of Sanya City Center, about 15 km, and west of the Ends of the Earth tourist attractions District, about 6 km. Sanya Bay seafront is nearby, only about 4.5 km.

TSIMF covers an area of 140 acres and a building area of nearly 30,000 square meters. It has a lecture hall, a variety of meeting rooms, libraries, parking lot, restaurants, playgrounds, gym, swimming pool and other amenities. Conference building has an area of 13,000 square meters, with more than 10 large and small conference rooms to meet different meeting requirements.

# Schedule

## The Schedule of 2023 Homological Algebra of the Infrared

Beijing Time	Mon 01.09	Tue 01.10	Wed 01.11	Thu 01.12	Fri 01.13
08:30-09:30 am		Ludmil Katzarkov	Greg Moore	Ahsan Khan	Si Li*
09:30-10:00 am	Tea Break				
10:00-11:00 am	Mikhail Kapranov	David Nadler	Jie Gu*	Yan Soibelman	Bohan Fang
03:30-04:00 pm	Tea Break				
04:00-05:00 pm	Alexey Bondal	Sibasish Banerjee	Johannes Walcher	Vadim Schechtman	Andrey Losev
05:15-06:15 pm	Timothy Logvinenko	Pietro Longhi	Maxim Kontsevich	Lev Soukhanov	Yu-Wei Fan

\*" is offline talk

**Please feel free  
to contact us  
if you have any requirements  
如有特殊情况请及时联系**

### Beijing 北京 (YMSC)

Yuezhao Zhang 张玥钊: 158 1108 0773

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### Sanya 三亚 (TSIMF)

Airport pick-up and nucleic acid testing matters 接送机及核酸检测事宜

Arrangement of rooms and meals 食宿安排

Miss Li Ye 叶莉: 139 7679 8300

Network and website 网络、设备及网页浏览

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# Summary

Jan. 9<sup>th</sup> Monday 

## Title: Algebra of the Infrared, Fourier transform and resurgence

**Beijing time** 10:00-11:00 am

**Speaker** Mikhail Kapranov (mikhail.kapranov@protonmail.com)

**Abstract:** A part of the Algebra of the Infrared can be interpreted as a categorification of the study of the Fourier transform for perverse sheaves (or holonomic regular D-modules) on the complex line  $\mathbf{C}$ . The talk will summarize this approach, developed in an earlier work with Y. Soibelman and L. Soukhanov and discuss a possible generalization to the context of resurgence. This generalization involves perverse sheaves on  $\mathbf{C}$  which are algebras with respect to additive convolution. Such perverse sheaves can have infinitely many singularities.

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## Title: Noncommutative resolutions of singularities and schobers

**Beijing time** 04:00-05:00 pm

**Speaker** Alexey Bondal (bondal@mi-ras.ru)

**Abstract:** We discuss the relation of the null-category of noncommutative resolutions with relative singularity categories and application to relevant schobers.

# Summary



Jan. 9<sup>th</sup> Monday

## Title: Skein-triangulated representations of generalised braids

**Beijing time** 05:15-06:15 pm

**Speaker** Timothy Logvinenko (LogvinenkoT@cardiff.ac.uk)

**Abstract:** The braid group  $Br_n$  encodes configurations of  $n$  non-touching vertical strands (“braids”) up to continuous transformations. There are many examples where  $Br_n$  acts on the derived category of an algebraic variety: the minimal resolutions of Kleinian singularities, the cotangent bundles of flag varieties, etc.

In this talk, I introduce a new structure: the category  $GBr_n$  of generalised braids. These are the braids whose strands are allowed to touch in a certain way. For triangulated categories, it is natural to impose certain relations which result in the notion of a skein-triangulated representation of  $GBr_n$ . These relations generalise the famous skein relation used to define oriented link invariants such as Jones polynomial.

We give two examples of skein-triangulated actions of  $GBr_n$ : on the cotangent bundles of varieties of full and partial flags in  $C^n$  and on categorical nil-Hecke algebras. The latter example shows that any categorical action of  $Br_n$  can be lifted to a skein-triangulated action of  $GBr_n$ , generalising a result of Ed Segal for  $n=2$ . This is a joint work with Rina Anno.

# Summary

Jan. 10<sup>th</sup> Tuesday 

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## Title: New Birational Invariants

**Beijing time** 08:30-09:30 am

**Speaker** Ludmil Katzarkov (lkatzarkov@gmail.com)

**Abstract:** In this talk we will offer new birational invariants connected with Non commutative Hodge structures. Examples will be considered.

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## Title: Morse theory for sheaves of categories

**Beijing time** 10:00-11:00 am

**Speaker** David Nadler (denadler@gmail.com)

**Abstract:** I will take the opportunity to describe some arguments appearing in joint work with Penghui Li and Zhiwei Yun on the cocenter of the affine Hecke category. We use some elementary aspects of Morse theory for sheaves of categories on a manifold. I will describe what we encounter, how we handle it, and raise some questions motivated by it.

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## Title: BPS state counting with Exponential networks

**Beijing time** 04:00-05:00 pm

**Speaker** Sibasish Banerjee (sibasishbanerjee@live.in)

**Abstract:** I will describe the framework of Exponential Networks for computing the BPS spectrum of M-theory on a local Calabi-Yau threefold times  $\mathbb{R}^4 \times S^1$ . Exponential networks define the counting of special Lagrangians in the mirror Calabi-Yau threefold. Therefore, from the geometric data of mirror curves, we propose how to compute the related DT invariants. I will briefly sketch how this is achieved for some concrete examples and discuss some relations with BPS quivers.

# Summary



**Jan. 10<sup>th</sup> Tuesday**

## **Title: Counting Lagrangian A-branes with networks**

**Beijing time** 05:15-06:15 pm

**Speaker** Pietro Longhi (longhip@phys.ethz.ch)

**Abstract:** The framework of spectral networks was introduced in physics as a way to compute BPS states of 4d  $N=2$  gauge theories. In this talk I will review a generalization, known as exponential networks, which produces enumerative invariants associated to special Lagrangians in certain Calabi-Yau threefolds. Applications include the computation of the exact spectrum for the mirror of the local Hirzebruch surface. I will also sketch a new derivation of this framework, which elucidates the geometric meaning of the invariants in terms of elementary data of A-branes.

# Summary

Jan. 11<sup>th</sup> Wednesday 

## **Title: 2d Categorical Wall-Crossing With Twisted Masses And An Application To Knot Invariants**

**Beijing time** 08:30-09:30 am

**Speaker** Gregory W. Moore (gwmoore@physics.rutgers.edu)

**Abstract:** For details, please refer to <http://www.physics.rutgers.edu/~gmoore/SoibelmanSeminar-September22-2021.pdf>

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## **Title: Resurgent quantum periods and BPS invariants**

**Beijing time** 10:00-11:00 am

**Speaker** Jie Gu (ej.ug.phys@gmail.com)

**Abstract:** Quantum periods appear in many contexts, from quantum mechanics to local mirror symmetry. They can be described in terms of topological string free energies and Wilson loops, in the so-called Nekrasov–Shatashvili limit. We consider the trans-series extension of the holomorphic anomaly equations satisfied by these quantities, and we obtain exact multi-instanton solutions for these trans-series. Building on this result, we propose a unified perspective on the resurgent structure of quantum periods. We show for example that the Delabaere–Pham formula, which was originally obtained in quantum mechanical examples, and which is similar to the Kontsevich–Soibelman transformation for the wall-crossing formulas of BPS invariants, is a generic feature of quantum periods, and therefore the Stokes constants of quantum periods could be interpreted as BPS invariants. We illustrate our general results with explicit calculations for the quantum mirror curve of local  $P^2$ .

# Summary

 **Jan. 11<sup>th</sup> Wednesday**

## **Title: Exponential Networks for Linear Partitions**

**Beijing time** 04:00-05:00 pm

**Speaker** Johannes Walcher ([walcher@uni-heidelberg.de](mailto:walcher@uni-heidelberg.de))

**Abstract:** Previous work has given proof and evidence that BPS states in local Calabi-Yau 3-folds can be described and counted by exponential networks on the mirror curve. The depictive representation is appealing for its elementarity, but so far only a handful of examples have been successfully worked out in detail. In this talk, we will exhibit an explicit correspondence between torus fixed points of the Hilbert scheme of points on  $C^2 \subset C^3$  and anomaly free exponential networks attached to the quadratically framed pair of pants. (Joint work with S. Banerjee, M. Romo and R. Senghaas.)

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## **Title: Non-holomorphic deformations of Landau-Ginzburg models**

**Beijing time** 05:15-06:15 pm

**Speaker** Maxim Kontsevich ([maxim@ihes.fr](mailto:maxim@ihes.fr))

**Abstract:** With a non-singular complex variety  $Y$  together with a holomorphic function  $W$  one can associate a sheaf of differential graded Lie algebras on  $Y$  consisting of polyvector fields with the differential given by the commutator with  $W$ . The formal germ of the derived moduli space is smooth and finite-dimensional if the critical locus is compact,  $Y$  is a Kähler and carries a non-vanishing holomorphic volume element. A part of the derived moduli space can be interpreted as moduli of holomorphic deformations of the pair  $(Y, W)$ , but the whole derived moduli space does not have a direct holomorphic interpretation. Such spaces are of great interests as they carry a weakened form of the Frobenius manifold structure, and are expected to describe genus 0 Gromov-Witten invariants for general symplectic manifolds. I'll talk about Fukaya-theoretic and Hodge-theoretic aspects of non-holomorphically deformed LG models, including a generalization of theory of spectra for isolated singularities.

# Summary

Jan. 12<sup>th</sup> Thursday 

## Title: Three Possible Generalizations of the Algebra of the Infrared

**Beijing time** 08:30-09:30 am

**Speaker** Ahsan Khan (khan@ias.edu)

**Abstract:** The algebra of the infrared is a remarkable structure that describes certain aspects of massive two-dimensional  $N=(2,2)$  theories. It is interesting to consider whether similar structures may exist in other physical contexts. In this talk, I will present three different contexts where we may expect such a structure. The first comes from considering a more general kind of two-dimensional  $N=(2,2)$  theory; one that allows for non-trivial twisted masses, leading to the possibility of particles in addition to solitons. The second concerns the study of four-dimensional  $N=2$  theories and their BPS particles. Finally, I will discuss some ideas relating to three-dimensional  $N=4$  theories and hyperKähler moment maps.

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## Title: Wall-crossing structures and Chern-Simons theory

**Beijing time** 10:00-11:00 am

**Speaker** Yan Soibelman (soibel@math.ksu.edu)

**Abstract:** The algebra of the infrared gives rise to a categorification of the Picard-Lefschetz wall-crossing formulas. Some wall-crossing formulas control the behavior of finite-dimensional exponential integrals. More general notion of wall-crossing structure was introduced 10 years ago by Kontsevich and myself. Recently we explained how it was related to the resurgence properties of many generating series in mathematics and physics. In this talk I am going to explain a speculative approach to the wall-crossing structure in complexified Chern-Simons theory. In the end it should give a conceptual explanation of analytic properties of the perturbative expansions in Chern-Simons theory.

# Summary

 **Jan. 12<sup>th</sup> Thursday**

## **Title: Vanishing cycles and Dold - Kan correspondence**

**Beijing time** 04:00-05:00 pm

**Speaker** Vadim Schechtman (vschechtman@gmail.com)

**Abstract:** We will be discussing some analogies between vanishing cycles and normalized chains appearing in the Dold - Kan correspondence between simplicial objects and chain complexes in an abelian category.

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## **Title: Algebra of the infrared and real Morse theory**

**Beijing time** 05:15-06:15 pm

**Speaker** Lev Soukhanov (merlin@solcery.xyz)

**Abstract:** I will talk about the occurrence of the Algebra of the Infrared structure in the different geometric setting: the solutions of a pair of commuting gradient vector fields.

# Summary

Jan. 13<sup>th</sup> Friday 

## Title: Elliptic chiral homology and chiral index

**Beijing time** 08:30-09:30 am

**Speaker** Si Li (sili@mail.tsinghua.edu.cn)

**Abstract:** We present an effective quantization theory for chiral deformation of two-dimensional conformal field theories. We explain a connection between the quantum master equation and the chiral homology for vertex operator algebras. As an application, we construct correlation functions of the curved beta-gamma/b-c system and establish a coupled equation relating to chiral homology groups of chiral differential operators. This can be viewed as the vertex algebra analogue of the trace map in algebraic index theory. The talk is based on the recent work [arXiv:2112.14572](https://arxiv.org/abs/2112.14572) [math.QA].

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## Title: Mirror curves and homological mirror symmetry for toric CY 3-orbifolds

**Beijing time** 10:00-11:00 am

**Speaker** Bohan Fang (bohanfang@gmail.com)

**Abstract:** For the mirror curve of a toric Calabi-Yau 3-orbifold, one can assign a Fukaya-type category. This category is modeled on the category of constructible sheaves on the 1-dim Lagrangian core of the mirror curve. They form a cosheaf and also have some sheaf properties, allowing us to show the equivalence to the matrix factorization category for the toric CY 3-orbifold by gluing, achieving a version of homological mirror symmetry. This talk is based on the joint work with Qingyuan Bai.

# Summary

 **Jan. 13<sup>th</sup> Friday**

## **Title: Towards counting of toric surfaces in complex manifolds through the Algebra of the Infrared**

**Beijing time** 04:00-05:00 pm

**Speaker** Andrey Losev ([aslosev2@yandex.ru](mailto:aslosev2@yandex.ru))

**Abstract:** It is well-known how effective WDVV equations are in counting rational curves in complex manifolds. The complex dimension 2 analogue of rational curves are toric surfaces. So, is it possible to get dim 2 version of WDVV?

In the first part of my talk I would explain why WDVV is a refinement of Bott-Morse equation  $D^2=0$ . In the second I will go to complex dimension 2 and will relate holomorphic maps of toric surfaces to a theory what Lev Soukhanov and I are calling 2-Morse theory. This theory studies 2-trajectories, i.e. Maps from  $\mathbb{R}^2$  to the target manifold along the pair of commuting vector fields. Evolution along commuting.

In the last part I will present the analogue of Morse-Bott equations for generating function for 2-trajectories passing through some cycles in the target space. I will explain the work of Soukhanov who showed that the 2-analogue of Morse equation is the Maurer-Cartan equation in the Algebra of the Infrared. Thus, we will stop one step from getting the 2-analogue of WDVV, that we still do not know.

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## **Title: On classification of autoequivalences of certain Calabi-Yau categories**

**Beijing time** 05:15-06:15 pm

**Speaker** Yu-Wei Fan ([yuweifanx@gmail.com](mailto:yuweifanx@gmail.com))

**Abstract:** The Nielsen-Thurston classification states that any mapping class of a Riemann surface is either of finite order, reducible, or pseudo-Anosov. Motivated by the analogy between Teichmüller theory and the theory of stability conditions on triangulated categories, we investigate the categorical analogue of the classification. First, we discuss the notion of pseudo-Anosov autoequivalences (joint work with Filip, Haiden, Katzarkov, and Liu). Second, for the finite order elements, we discuss the categorical Nielsen realization problems (ongoing work with Lai).





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